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AN ECONOMIC ANALYSIS OF JOINED COSTS AND BENEFICIAL EFFECTS OF WASTE RECYCLING

Much research has been done throughout the world to determine the economic and ecological profitability of waste utilization as a substitution for raw materials. Unfortunately, Poland has no sufficient solutions in this field. The deficiency of solutions impedes the work of groups of specialists in various fields involved with the rational planning of recycling. This research seeks to offer a simple unified framework to study the key economic features of the use of waste as substitute for nonrenewable resources in production in Poland and other countries. The purpose of this paper is to develop a model to obtain some key insights about the cost-effectiveness and define certain parameters which determine the effectiveness of production plants dedicated for processing waste into useful products. A plant's productive possibilities can be described by the production function which is determined empirically. In this model, the emphasis is put on plant functioning and is dependent on the amount of work destined for the production in fixed units, technical maintenance costs of the work fixture in fixed units (e.g. machines', cars' operating costs etc.), and the amount of modified waste. In order to calculate these costs, we need to consider the issue of precise measuring the overhead costs in order to maximize profitability of this kind of measures allocation so that the profit will be the highest. The task of measuring overhead (costs) comes down to finding the maximum of the production function. This research presents the mathematical model and provides some key insights about the profitability of the plants dedicated for processing waste into secondary materials. In this research, we also determined the maximum of the profit function and assess when the recycling plant will bring the profit and when it is the highest.

1. INTRODUCTION

We live in a socially interconnected world as well as in one displaying environmental interdependence. In most cases, human desires to use the Earth's available resources, including environmental resources, exceed the capacity of these resources to satisfy human wants completely [1]. From an economic viewpoint, developing

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countries such as Poland are heavily dependent on living resources and natural environments. Furthermore, developed countries benefit substantially from the conservation of their own resources through recycling/reuse.

The analysis of waste utilization as substitute natural resources has largely evolved as separate branches in economic literature. A notable exception is *On Natural Resources Substitution*" by Andre and Creda, presenting a simple dynamic model to gain some key insights about the substitution of renewable resources for non-renewable resources, and the consequences for sustainability [2]. Ready and Ready presented a dynamic model according to which the price of landfill disposal increases from the time a new landfill is opened until it is filled, then decreases slightly as a new landfill is opened, and then begins increasing once again, due to the problem of land scarcity [3]. Therefore, it is economical to start recycling some of the waste before the price of landfill disposal reaches that of recycling, in order to reduce the costs of landfill over time. In his pioneering work, Vivian [4] studied the costs and benefits of the current practice of dumping the construction waste into landfills and extracting new natural materials for new concrete production, as well as the proposed concrete recycling method to recycle the construction waste as aggregate for new concrete production. With the advent of the cost on the current practice, it is found that the concrete recycling method can result in huge savings.

The benefits gained from the concrete recycling method can balance the cost expended for the current practice. Therefore, recycling concrete waste for new production is a cost-effective method that also helps to protect the environment as well as to achieve construction sustainability.

Towards the end of the 20th century, and at the beginning of the 21st century, the problem of using waste as a substitute for primary materials has become one of the basic issues of environmental protection and waste management in the world. Much research on recycling has been done by such authors as Rhyner [5], Sharma et al., [6], di Vita [7, 8], Fletcher and Mackay [9], Hsu and Kuo [10] Nakamura [11, 12], Berglund [13], Haque et al. [14] and Alwaeli [15, 16]. The authors analysed various microeconomic models in which the economic profitability from waste recycling and certain ways of reducing the current value and future costs connected with waste management were considered.

Over the last few decades, consumption behaviour in Poland has changed dramatically in accordance with rapid economic growth. Waste disposal caused serious problems. Altogether more than 1.74 milliard tons of waste have been landfilled in Poland, and this amount increases approximately by 133 million tons every year [17]. Landfilling is still the main disposal method in Poland, although other methods are used on a negligible scale. Recycling is the most effective method for solving this problem because it not only reduces amount of waste, but also mitigates the depletion of natural resources resulting from economic development.

2. THE MODEL

The cost-effectiveness of waste recycling depends on a variety of inherent expenses. These costs include the expenses borne by the waste supplier, namely the cost of obtaining waste, transporting it to the segregation plants, and finally, the cost of the actual waste segregation. By contrast, recycling plants which use waste as raw materials bear the costs of waste processing as well as their clearance. In addition, we need to take into consideration those costs which are associated with amortization, equipment maintenance, waste storage, and labour. This work emphasizes the economic profitability of the plants dedicated for processing waste into secondary materials. The economic cost-effectiveness of the plant dedicated for processing waste into secondary materials can be defined as a production function. From economic experience [2, 7, 18] it seems that the production function has the form:

$$f(x_1, x_2, x_3, x_4) = a x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3} x_4^{\alpha_4} \tag{1}$$

In which coefficients a, α_1, α_2, α_3, α_4, are empirically matched and fulfil the following conditions:

$$\alpha > 0, \quad \alpha_2 > 0, \quad \alpha_3 > 0, \quad \alpha_4 > 0, \quad \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 < 1$$

The production function depends on the amount of work destined for the production in fixed units (x_1), the technological maintenance costs of the work fixture in fixed units (x_2), the amount of the modified raw material (x_3), and the amortization costs (x_4), etc.

Let us denote unit costs of particular elements of the production by q_1, q_2, q_4, q_4,. This means that the work unit costs q_1 ZLP, technological maintenance unit of the work fixture costs q_2 ZLP, etc. The unit of the product has the price of γZLP.

Having available x_1, x_2, x_3, x_4, the production plant will produce $f(x_1, x_2, x_3, x_4)$ of the product units in a specific time unit (e.g. one month).

Owing to the fact that we have K means destined for the production in a given time, we get the condition that the sum of the input designated for the plant exploitation will not exceed K reduced by the permanent costs K_1, (maintenance of both administration and plant). The aforementioned costs must be borne despite the volume of the production.

$$q_1 x_1 + q_2 x_2 + q_3 x_3 + q_4 x_4 \le K - K_1 \tag{2}$$

The left side of the inequality stands for the relative sum of the plant, whereas the right side signifies the amount of measures available.

We need to consider the issue of measuring overhead costs in order to ensure the highest possible profit. The profit from the production is expressed by the following formula:

$$\Phi(x_1,x_2,x_3,x_4)=\gamma f(x_1,x_2,x_3,x_4)-(q_1x_1+q_2x_2+q_3x_3+q_4x_4) \tag{3}$$

Because of the fact that the function increases with regard to each variable, the whole capital which we have at our disposal, must be divided. The following conclusion can be drawn:

$$x_1>0, \quad x_2>0, \quad x_3>0, \quad x_4>0, \quad x_1+x_2+x_3+x_4=K-K_1 \tag{4}$$

The task of measuring overhead costs comes down to finding the maximum of the function (3) taking into account the aforementioned conditions (4). The mathematical solution to the issue of optimization will be discussed later.

Assuming that $(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4)$ is the solution to the problem, the profit from the plant operating in a specific time will be:

$$\Phi\left(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4\right)=\gamma f\left(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4\right)-(K-K_1)$$

However, the profit must fulfil certain economic conditions.

The value of the plant amounts to Π ZLP, the interest on the financial market in a fixed time unit comes to p%, total depreciation of the working place amounts to q%. It may be concluded that the profit from the plant exploitation must be higher. Eventually, it will come to:

$$\Phi(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4)-\Pi\cdot\frac{p}{100}\cdot\frac{q}{100}>0 \tag{5}$$

The profit must be positive in order for the plant to have economic profitability.

The profit issue of optimization will be solved based on the Kuhn–Tucker (K–T) methods. From the K–T condition, if the system of equations

$$\frac{\partial L}{\partial x_i}=0, \qquad i=1, 2, \ldots, n$$

$$\frac{\partial L}{\partial \lambda}=0$$

has a positive solution

$$x_i>0, \qquad i=1, 2, \ldots, n, \qquad \lambda>0$$

in which

$$L(x,\lambda)=\gamma f(x)-\lambda\left(q_1x_1+q_2x_2+\ldots+q_nx_n-K\right)$$

is the Lagrange function, and the function for $f(x)$ is a concave one which means that the solution of the system of Eqs. (5) is the solution to the problem of the maximization of the profit function

$$\Phi(x) = \gamma f(x) - K$$

Nevertheless, there is the condition

$$q_1 x_1 + q_2 x_2 + \ldots + q_n x_n = K$$

Hence, we have the system of equations to be solved

$$\frac{\partial L}{\partial x_i} = \gamma \frac{\partial f}{\partial x_i} - \lambda q_i = 0, \qquad i = 1, 2, \ldots, n$$

$$\frac{\partial L}{\partial x} = q_1 x_1 + q_2 x_2 + \ldots + q_n x_n - K = 0$$

If the solution of this system is positive, it is a point in which the profit function reaches a maximum,

The profit function obtained from the production amounts to:

$$Z(x) = \gamma f(x) - K \tag{6}$$

where γ is the unit cost of the produced product, K – the amount of money destined for the production in a specific time (e.g. one month).

The function $f(x)$ is the productive function depending on the means of production and is expressed as

$$f(x) = a x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n} \tag{7}$$

where:

$$x = (x_1, \ldots, x_n) \in R_+^n$$

$$\alpha_i > 0, \qquad i = 1, 2, \ldots, n, \qquad \sum_{i=1}^{n} \alpha_i = a < 1$$

The function (7) is the concave one.

We determine the maximum of the function $Z(x)$ having the given condition

$$q_1 x_1 + q_2 x_2 + \ldots + q_n x_n = K, \qquad x_i > 0, \qquad i = 1, 2, \ldots, n \tag{8}$$

in which $(q_1, q_2, \ldots, q_n)$ is the vector of the unit prices of the means of production $x \in R_+^n$.

The Lagrange function

$$L(x, \lambda) = \gamma f(x) - \lambda(q_1 x_1 + q_2 x_2 + \ldots + q_n x_n - K), \qquad x \in R_+$$

and the Kuhn–Tucker function should be applied

$$\left\{\begin{array}{l} \dfrac{\partial L}{\partial x_i} = \gamma \dfrac{\partial f}{\partial x_i} - \lambda q_i = 0, \qquad i = 1, 2, ..., n \\ \dfrac{\partial L}{\partial \lambda} = q_1 x_1 + q_2 x_2 + \cdots + q_n x_n - K = 0 \end{array}\right\} \tag{9}$$

If the system has only one solution and $L(x, \lambda)$ is the concave function, the solution is the maximum of the function given by Eq. (1). However, the condition in Eq. (3) must be fulfilled.

We determine the solution of the system depending on the parameter γ. Because of the fact that

$$\frac{\partial f}{\partial x_i} = a x_1^{\alpha_1} ...(\alpha_i x_i^{\alpha_i - 1})... x_n^{\alpha_n} = \frac{\alpha_i}{x_i} f(x)$$

the system of Eqs. (9) is represented by:

$$\gamma \frac{\alpha_i}{x_i} f(x) - \lambda q_i = 0, \qquad i = 1, 2, ..., n$$

For $i \neq j$ we have

$$\gamma \frac{\alpha_i}{x_i} f(x) = \lambda q_i \tag{10a}$$

$$\gamma \frac{\alpha_j}{x_j} f(x) = \lambda q_j \tag{10b}$$

Dividing Eq. (10a) by (10b), we obtain

$$\frac{\frac{\alpha_i}{x_i}}{\frac{\alpha_j}{x_j}} = \frac{q_i}{q_j}, \qquad \frac{\alpha_i x_j}{\alpha_j x_i} = \frac{q_i}{q_j}, \qquad \frac{x_j}{\frac{\alpha_j}{q_j}} = \frac{x_i}{\frac{\alpha_i}{q_i}}$$

Hence, the vectors

$$\left(x_1, x_2, ..., x_n\right) \qquad \text{and} \qquad x\left(\frac{\alpha_1}{q_1}, \frac{\alpha_2}{q_2}, ..., \frac{\alpha_n}{q_n}\right)$$

have proportional coordinates; in consequence, we obtain

$$x_i = t \frac{\alpha_i}{q_i}, \qquad i = 1, 2, ..., n$$

where t is a parameter.

After introducing the obtained values into Eq. (8) we get

$$q_1 t \frac{\alpha_1}{q_1} + q_2 t \frac{\alpha_2}{q_2} + \ldots + q_n t \frac{\alpha_n}{q_n} - K = 0$$

The conclusion is that

$$t(\alpha_1 + \alpha_2 + \ldots + \alpha_n) = K$$

Eventually,

$$t = \frac{K}{\alpha}$$

Consequently, we get the solution of the systems Eqs. (9) and (5)

$$\bar{x}_i = \frac{K}{\alpha} \cdot \frac{\alpha_i}{q_i}, \qquad i = 1, 2, \ldots, n$$

which is the solution of the optimum task.

The maximum profit, if we take into consideration investing *K* capital, amounts to:

$$\tilde{Z}(K) = Z(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) = \gamma a (\frac{K}{\alpha}\frac{\alpha_1}{q_1})^{\alpha_1} \cdot (\frac{K}{\alpha}\frac{\alpha_2}{q_2})^{\alpha_2} \cdot \ldots \cdot (\frac{K}{\alpha}\frac{\alpha_n}{q_n})^{\alpha_n} - K$$

$$= \gamma a (\frac{K}{\alpha})^{\alpha_1 + \alpha_2 + \ldots + \alpha_n} (\frac{\alpha_1}{q_1})^{\alpha_1} (\frac{\alpha_2}{q_2})^{\alpha_2} \ldots (\frac{\alpha_n}{q_n})^{\alpha_n} - K = \gamma (\frac{K}{\alpha})^{\alpha} f(\frac{\alpha_1}{q_1}, \frac{\alpha_2}{q_2}, \ldots \frac{\alpha_n}{q_n}) - K$$

Marking the constant

$$\gamma f\left(\frac{\alpha_1}{q_1}, \frac{\alpha_2}{q_2}, \ldots, \frac{\alpha_n}{q_n}\right) = R,$$

we get

$$\tilde{Z}(K) = \left(\frac{K}{\alpha}\right)^{\alpha} R - K$$

Now, we need to study how much money we have to invest in the production in order to achieve the maximuml profit. For this purpose, the variability of function $\tilde{Z}(K)$ must be studied

$$\tilde{Z}'(K) = \frac{R}{\alpha^{\alpha}} \alpha K^{\alpha - 1} - 1 = 0$$

$$R\alpha^{\alpha - 1} = K^{1 - \alpha}$$

$$\tilde{K} = \alpha R^{1/(1-\alpha)}$$

$$\tilde{Z}(\tilde{K}) = (R^{1/(1-\alpha)})^{\alpha} R - \alpha R^{1/(1-\alpha)} = R^{1/(1-\alpha)} - \alpha R^{1/(1-\alpha)} = (1-\alpha) R^{1/(1-\alpha)}$$

Based on the fact that $Z(K) = 0$ for $K = 0$ and $K = R^{1/(1-\alpha)}/\alpha^{1/(1-\alpha)}$, the course of the function $Z(K)$ is as shown in Fig. 1.

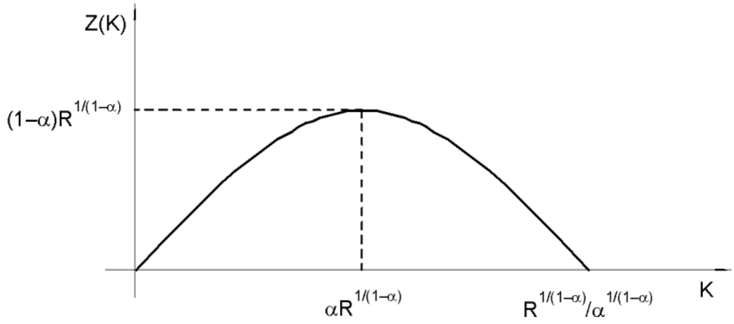

Fig. 1. Profit dependence on the invested capital

The unconditional extreme of the function (6), i.e. without the condition (8), is seen in the point

$$\bar{x}_i = R^{1/(1-\alpha)} \frac{\alpha_i}{q_i}, \qquad i = 1, 2, ..., n$$

For $K > \alpha R^{1/(1-\alpha)}$, it is uneconomic to invest K funds due to the fact that the profit from the production will be smaller than for the unconditional extreme.

3. DISCUSSION

It is a challenge to reduce the increasing flow of waste. The development of an efficient waste recycling approach will help us to explore new opportunities for urban and environmental protection. Despite the fact that properties of the primary materials have been lost, the waste still carries both the value of subjective human work as well as the energy used for their production. This waste constitutes a potential source of secondary materials and fuels. Recycling is the most effective method for solving these problems because the recycling process not only reduces amount of waste, but also mitigates the depletion of natural resources resulting from economic development. Additionally, recycling reduces the cost of transportation and, relying on the analysis of the use of waste as a substitute in the production held by LCA, shortens, in some cases, the production process.

Owing to the fact that we have K meaning destined for the production in a given time, we get the condition that the sum of the input designated for the plant exploitation will not exceed K reduced by the permanent costs K_1, that is, the maintenance of both the administration and the plants.

Because of the fact that the production function increases with regard to each variable, the whole capital, which we have at our disposal, must be divided. We considered the issue of measuring and dividing overhead costs, in order that the highest profit can be achieved.

The purpose of this research is to present the mathematical model and to provide some key insights about the profitability of the plants dedicated for processing waste into secondary materials. The profit function obtained from the production amounts to $Z(x) = \gamma f(x) - K$ in which γ is the unit cost of the produced product, K is the quantity of money destined for the production in a specific time (e.g. one month) and $f(x)$ is the production function.

In this research, we also determined the maximum of the profit function. Based on Eq. (6), the maximum profit, implicate that if we take into consideration investing K capital, amounts to:

$$\tilde{Z}(K) = \left(\frac{K}{\alpha}\right)^{\alpha} R - K = K\left(\frac{R}{\alpha}\left(\frac{\alpha}{K}\right)^{1-\alpha} - 1\right)$$

This means that the profit is proportional to the sum of invested capital K multiplied by the factor $\left(\frac{R}{\alpha}\left(\frac{\alpha}{K}\right)^{1-\alpha} - 1\right)$ which is decreasing upon increasing K. Thus in order to have the maximum profit, we have to invest $K = R^{1/(1-\alpha)}/\alpha^{1/(1-\alpha)}$.

Based on Fig. 1, the recycling plant will bring the profit for $0 < K < \alpha R^{1/(1-\alpha)}$; nevertheless, the profit is the highest for $K = (\alpha R_2)^{1/(1-\alpha}$. While for $K > \alpha R^{1/(1-\alpha)}$ it is uneconomic to invest K fund due to the fact that the profit from the production will be smaller than for the unconditional extreme.

Much research has been done throughout the world to determine the economic and ecological profitability of utilization of secondary materials (waste) as a substitution for raw materials. Unfortunately, Poland does not have sufficient solutions in this field. The deficiency of solutions impedes the work of groups of specialists in various fields involved with the rational planning of recycling. These are the results of our mathematical model of economic profitability of secondary materials (waste) utilization as the substitute for primary materials. Currently, no empirical analyses have been carried out on this issue. This research seeks to offer a simple unified framework to study the key economic features of the use of waste as substitute for nonrenewable resources in production in Poland and other countries.

APPENDIX A

As a first attempt to evaluate the empirical performance of the model of the previous section, we will consider example in which the production function is as follows:

$$f(x_1, x_2) = x_1^{0,25} x_2^{0,25}$$

The profit function amounts to

$$Z(x_1, x_2) = P x_1^{0,25} x_2^{0,25} - q_1 x_1 - q_2 x_2$$

where: P is the unit price of the produced product, q_1, q_2 – unit prices of raw materials.

We need to determine the unconditional extreme of the function. We have

$$\frac{\alpha_1}{q_1} = \frac{1}{4q_1}, \qquad \frac{\alpha_2}{q_2} = \frac{1}{4q_2}, \qquad f\left(\frac{1}{4q_1}, \frac{1}{4q_2}\right) = \left(\frac{1}{4q_1}\right)^{0,25} \left(\frac{1}{4q_2}\right)^{0,25} = \frac{1}{2(q_1 q)^{0,25}}$$

$$R = \frac{P}{2(q_1 q_2)^{0.25}}$$

$$\bar{x}_1 = \left(\frac{P}{2(q_1 q_2)^{0,25}}\right)^2 \frac{1}{4q_1} = \frac{P^2}{16 q_1 (q_1 q_2)^{0,5}}, \qquad \bar{x}_2 = \left(\frac{P}{2(q_1 q_2)^{0,25}}\right)^2 \frac{1}{4q_2} = \frac{P^2}{16 q_2 (q_1 q_2)^{0,5}}$$

$$f(\bar{x}_1, \bar{x}_2) = \frac{P}{4\sqrt{q_1 q_2}}$$

$$Z(\bar{x}, \bar{y}) = \frac{P^2}{4\sqrt{q_1 q_2}} - \frac{P^2}{8\sqrt{q_1 q_2}} = \frac{P^2}{8\sqrt{q_1 q_2}}$$

We can conclude that if we invest the means of production $\bar{x}_1, \bar{x}_2$, we will obtain the production $f(\bar{x}_1, \bar{x}_2)$ whose profit amounts to $Z(\bar{x}, \bar{y})$.

APPENDIX B

Now we have to consider one more example in which the production function is as follows:

$$f(x) = \alpha_1 \ln x_1 + \alpha_2 \ln x_2 + \ldots + \alpha_n \ln x_n$$

where

$$x = (x_1, \ldots, x_n) \in R_+^n, \qquad \alpha_i > 0, \qquad i = 1, 2, \ldots, n$$

The function $f(x)$ is concave for $x \in R_+^n$.

The profit function

$$Z(x) = Pf(x) - K$$

where P is the price of the produced product, K the total sum of means destined for the production, i.e.

$$q_1x_1 + q_2x_2 + \ldots + q_nx_n = K$$

where q_i is the unit price of the used raw material.

We maximize the Lagrange function

$$L(x,\lambda) = \gamma f(x) - K - \lambda(q_1x_1 + q_2x_2 + \ldots + q_nx_n - K)$$

We solve the system of equations

$$\frac{\partial L}{\partial x_i} = P\frac{\alpha_i}{x_i} - \lambda q_i = 0, \qquad i = 1, 2, \ldots, n$$

$$\frac{\partial L}{\partial x} = q_1x_1 + q_2x_2 + \ldots + q_nx_n - K = 0$$

we have

$$\frac{P\alpha_i}{x_i} = \lambda q_i, \qquad i = 1, 2, \ldots, n$$

For $i \neq j$ we have

$$\frac{P\alpha_i}{x_i} = \lambda q_i, \qquad \frac{P\alpha_j}{x_j} = \lambda q_j$$

Dividing the equations by their members, we obtain

$$\frac{\alpha_i}{x_i}\frac{\alpha_j}{x_j} = \frac{q_i}{q_j}$$

Hence we get

$$\frac{x_j}{\frac{\alpha_j}{q_j}} = \frac{x_i}{\frac{\alpha_i}{q_i}}.$$

Therefore, the vectors

$(x_1, x_2, \ldots, x_n)$ and $x\left(\frac{\alpha_1}{q_1}, \frac{\alpha_2}{q_2}, \ldots, \frac{\alpha_n}{q_n}\right)$, are parallel.

In that case there exists a constant such that

$$x_i = t_i \frac{\alpha_i}{q_i}, \qquad i = 1, 2, ..., n$$

Substituting it into Eq. (8), we have

$$t\alpha_1 + t\alpha_2 + ... + t\alpha_n = K$$

Stating that

$$\alpha_1 + \alpha_2 + ... + \alpha_n = a$$

we obtain

$$ta = K, \qquad t = \frac{K}{a}$$

consequently

$$\bar{x}_i = \frac{K}{a}\frac{\alpha_i}{q_i}, \qquad i = 1, 2, ..., n$$

is the solution to the problem of maximization. We have

$$f(\bar{x}) = \alpha_1 \ln \frac{K}{a}\frac{\alpha_1}{q_1} + \alpha_2 \ln \frac{K}{a}\frac{\alpha_2}{q_2} + ... + \alpha_n \ln \frac{K}{a}\frac{\alpha_n}{q_n}$$
$$= (\alpha_1 + \alpha_2 + ... + \alpha_n)\frac{K}{a} + \alpha_1 \ln \frac{\alpha_1}{q_1} + \alpha_2 \ln \frac{\alpha_2}{q_2} + ... + \alpha_n \ln \frac{\alpha_n}{q_n} = a \ln \frac{K}{a} + f(\bar{x})$$

Therefore the maximum profit from the production process is

$$Z(\bar{x}) = P(a \ln \frac{K}{a} + f(\bar{x})) - K$$

Stating that $Pf(\bar{x}) = R$ we have

$$Z(\bar{x}) = Pa \ln \frac{K}{a} - K - R$$

We investigate which means have to be involved so that the profit is maximized. We have

$$\frac{dZ(\bar{x})}{dK} = \frac{Pa}{K} - 1 = 0$$

thus $K = Pa$.

Therefore, the maximum profit amounts to

$$\overline{Z} = Pa \ln P - Pa - R \qquad \text{for} \qquad \overline{x}_i = \frac{P\alpha_i}{q_i}, \qquad i = 1, 2, ..., n$$

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