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RATIONAL OPERATION OF VARIABLE DECLINING RATE FILTERS

An approximate solution to the system of equations governing the flow distribution among variable declining rate (VDR) filters results in flow rates through filters being elements of a geometrical progression. Based on this approximation, it was deduced how to operate a plant in order to keep the same flow rates through VDR filters for various total head losses of flow. These principles of operation were carefully verified using the accurate di Bernardo mathematical model of VDR filter plants. It was deduced that the longest filter runs result from such an operation of a plant for which the ratio of the highest to the average flow rates through a filter and simultaneously the affordable total head loss of flow through the plant are the highest.

1. INTRODUCTION

In the forty years since the first application of the variable declining rate (VDR) control system [1], many existing plants have been installed with orifices to replace expensive and sometimes unreliable mechanical flow rate controllers. VDR filters are especially popular in South America, but several water treatment plants using the VDR control system operate successfully in the U.S.A., and in the European Community.

The idea of declining filtration might be advantageous in further application of activated carbon, either by itself or as one of filter media layers. A longer time is required for filtration in the case of a clogged activated carbon filter. This longer time facilitates improved adsorption of soluble organic compounds. However, if granular activated carbon (GAC) filters follow conventional sand or sand–anthracite filters the GAC media often requires backwashing to avoid high bacteriological content in filtrate, before any significant clogging of carbon is observed [2]. So far it is difficult to

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judge how effectively the VDR system of operation will compete with mechanical flow controllers, as this second type of a flow control system can also be constructed to operate under a declining flow rate in a more manageable fashion than that offered by orifices installed at the outflows from the filters. However, it is evident that variable declining rate (VDR) filters are an economically reasonable solution for all treatment plants unable to meet the water quality demands or those poorly controlled. Moreover, it has been shown several times that the quality of filtrate produced by filters operated under this system is at least as good as that obtained under a constant filtration velocity [3–8]. Thus, it is easy to understand why this invention has spread rapidly in countries where increasing water demands exist but still low financial capabilities. VDR filters offer some advantages as an increase in plant capacity, longer filter runs or a decrease in the total head loss in the system. If only one of these factors is chosen for maximization, it limits or excludes making use of other opportunities. Therefore, a proper design of a VDR filter system is best approached as an optimisation problem.

Optimisation theory is a powerful tool which deals with hundreds or even thousands of decision variables, especially when linear problems are concerned. In the case of retrofitting a plant with a VDR filter control system there are only a few operation parameters to be optimised. Therefore major difficulty results from a lack of comprehensive knowledge about relations existing between them. However, there are several prospective methods of computing hydraulic control systems of VDR filters. An approximate solution to di Bernardo's [9, 10] mathematical model is used here to elaborate several relationships between filters operating in a bank. Most of these relationships have been found to be useful in an optimisation procedure proposed for the hydraulic control system of a VDR filter plant. It was assumed that an existing water treatment plant is equipped with hydraulic flow rate controllers (orifices) located at the outflow pipe from each of the filters [10, 11]. The maximum available head loss, the ratio of the maximum to the mean flow rate, the flow rate through the whole plant and the filter media are assumed to be fixed. The coefficients characterizing the friction created by orifices, the ratio of maximum q_{max} to average q_{avr} flow rates through the filter units, the height of water surface fluctuation above filter media h_0, and the value of the total head loss of flow through the system H are decision variables. The goal is to minimize the frequency of backwash subject to the constraints imposed on the decision variables by the total plant capacity, affordable head loss of flow through the system, and the highest admissible flow rate through a clean filter.

2. FLOW CONTROL SYSTEM

Variable declining rate filters are equipped with identical orifices installed at inflows or outflows from all filters. The operation of such a filter plant is based on cooperation between head losses created by linear laminar flow through filter media with head loses of turbulent flow through orifices and transitional flow through the drainage [11, 12]. The operational parameters of the flow control system should be selected in such a way to restrict flow through the freshly backwashed filters mostly by head losses of flow through orifices. However, for the most significantly clogged filter, waiting for backwash, this head loss should be negligible in comparison with the head loss of flow through the filter media. The principles of the VDRF plant operation are described elsewhere [10–18]. The following assumptions were made:

- there are at least four filters in a bank,
- all filters are identical,
- inflows are located below the lowest water surface above the filter media,
- head losses in piping are negligible in comparison with those created by filter media,
- orifices are located at outflows from filters.

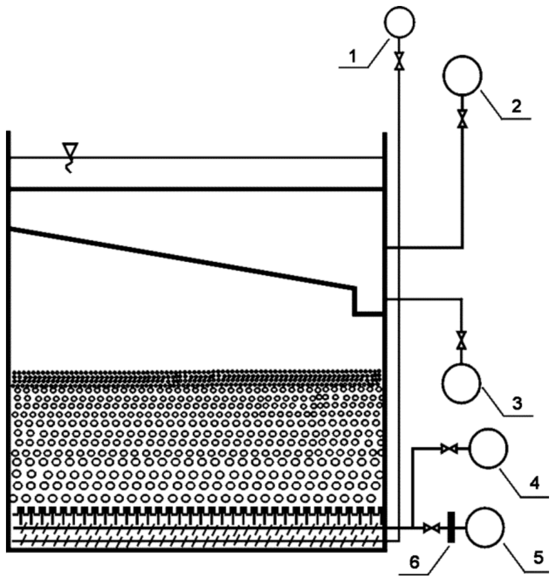

Fig. 1. The place of installation of an orifice:
1 – air, 2 – raw water, 3 – wastewater, 4 – backwash water, 5 – filtrate, 6 – orifice

Under these assumptions the unconfined water surface level is the same above all filters in any moment of time. This level rises in time due to clogging of filter media, to drop down quite rapidly after restoring the most recently backwashed filter to service. The place of installing orifices controlling flow rates through filters is presented in Fig. 1. Examples of patterns of water and flow rates are presented in Figs. 2, 3.

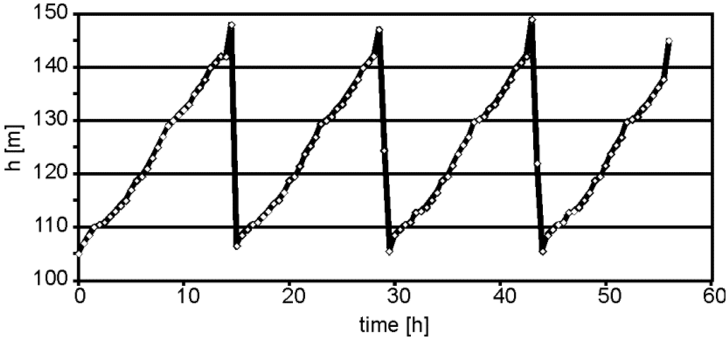

Fig. 2. An example of water table fluctuations at the pilot VDR filter plant constructed at the Cracow University of Technology

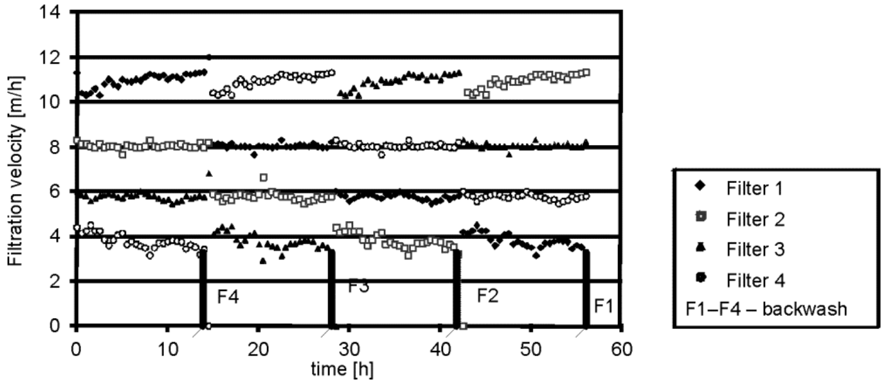

Fig. 3. An example of flow rates recorded at the pilot VDR filter plant constructed at the Cracow University of Technology

3. EQUATIONS

Di Bernardo [9, 10] proposed a model of the variable declining rate filter control system under the following assumptions:

1. All flow rates vary only at time of and just after a backwash of one filter, thus they remain essentially constant between subsequent backwashes in a plant.

2. The period of a backwash of one of the filters is so short that the hydraulic resistances of the filters remaining in service are not visibly affected by clogging during this period (the resistance of a filter *i* is recognized to be the same before and after the backwash of a filter *z*).

Di Bernardo [9, 10] compared theoretical results obtained by his model with experimental data collected from a pilot plant as well as from a full-scale filtration plant and he received strong support for his theory. Dąbrowski [11] found the Di Bernardo model to give results similar to those produced from the set of equations elaborated by

Arboleda et al. [16] and summarized di Bernardo's model in a set of $z + 1$ Eqs. (1)–(3). Mackie et al [12] proved the solution to the system of Eqs. (1)–(3) to be close to results of computations based on the unit bed element (UBE) [19] model of a filter plant

- $z - 1$ equations

$$\frac{H - h_0 - c_2 q_{i+1}^n}{q_{i+1}} = \frac{H - c_2 q_i^n}{q_i} \tag{1}$$

- 1 equation

$$H - h_0 = c_1 q_1 + c_2 (q_1)^n \tag{2}$$

- 1 equation

$$Q = \sum_{i=1}^{i=z} q_i \tag{3}$$

where: c_1 is a proportional coefficient characterizing the resistance of clean medium, c_2 – coefficient of turbulent head losses, H – total head loss of flow through the plant just before a backwash, h_0 – height of water surface increase between backwashes, i – number of a filter in a bank, n – exponent of turbulent and transitional head loss created by orifices and drainage, q_i – flow rate through ith unit, $q_i = q_1$ for $i = 1$, etc., Q – total flow rate through the system, z – number of filters in a bank.

Equations (1) does not account for compressing of deposit in filter media, which is an easily visible phenomenon in the case of increasing [20] and less visible for decreasing [21] of coagulated water suspension filtration velocity. However, extensive pilot plant experiments [22] proved that the di Bernardo model [10] was surprisingly accurate for modelling filter plants following chemical pretreatment and flocculators. There are $(z - 1)$ equations (1) and two single Eqs. (2), (3) where z is the number of filters in a bank.

Three approximate solutions to the system of Eqs. (1)–(3) were elaborated [11] and tested in comparison with accurate numerical solutions and with experimental data. Approximation (4) to Eq. (1) was definitely of the worst accuracy but still gave quite acceptable results:

$$\frac{q_{i+1}}{q_i} = 1 - \frac{h_0}{H} \tag{4}$$

According to this approximation flow rates through filters are elements of a geometrical progression. Obviously approximated Eq. (4) to Eq. (1) may be applied only as long as the parameters of a filter plant operation H, h_0, z, c_1, c_2, q_1 ..., q_i ..., .q_z are in the range commonly used in water filtration practice. Beyond this range the substitution of Eq. (1) by Eq. (4) may result in unexpectedly high errors of calculations.

4. BASIC RULES

Some general properties of VDR filters developed from a solution to the set of Eqs. (2)–(4) give an idea which particular parameters should be considered in optimisation of filter runs. Since Eq. (4) is a rough approximation to Eq. (1), thus all the conclusions developed here were carefully verified in numerical experiments by rigorous solution to the set of Eqs. (1)–(3). The theory developed here is limited to a reconstruction of an existing plant, so the coefficient c_1, describing the hydraulic resistance of a clean porous medium against the flow, is given. The head loss H, height of water table fluctuations over the filters h_0, and the turbulent head loss coefficient c_2 are adjusted in calculations to meet the required optimisation goals. However, prior to formulation of the optimisation problem, some properties of the solution to the system of Eqs. (2)–(4) are summarized. These properties will be used later in search for a special family of solutions to the set of Eqs. (1)–(3) for which flow rates through filter units remain almost the same when parameters of a filter plant operation change according to basic rules defined in the next paragraph.

5. PROPERTIES OF THE SOLUTION TO THE SYSTEM OF EQUATIONS (2)–(4)

Prior to formulation of the optimisation problem, some properties of the solution to the system of Eqs. (2), (3), (4) are summarized. Property 1 follows directly from Eq. (4).

Theorem 1. The ratio of q_{i+1}/q_i does not depend on the number i of a filter in a bank. If the values of H and h_0 are the subject of change in such a particular way that h_0/H is the same each time the ratio q_{i+1}/q_i remains constant.

From this statement and from Eqs. (2), (3) a practical rule for management of VDR control systems may be directly deduced.

Theorem 2. In order to receive the same flow rates through all filters in a plant $q_1, ..., q_i, ..., q_z$ for any feasibly chosen new values of the total head loss of flow through a plant before a subsequent backwash H, parameters of the plant operation c_2, h_0 should be adjusted according to the following rules:

1) h_0 is to be calculated for a new value H directly from a constant (equal to the previous) value of the quotient h_0/H.

2) The coefficient characterizing the orifice c_2 is to be calculated from Eq. (2) in order to give the same q_1.

Finally, a whole family of solutions to the set of Eqs. (2)–(4) characterized by the same total plant capacity Q and the same flow rates q_i through all filters is obtained.

Such a family of solutions is presented in Fig. 4 based on a rigorous numerical solution to the system of Eqs. (1)–(3) supplied with the same data c_1, Q, z, n as given in di Bernardo's [9, 10] example of calculations for a pilot plant – see Table 1. In these computations, the ratio of q_1/q_{avr} was imposed to be equal to 1.3. The flow rates now refer to 1 m^2 of a filter (hydraulic load), so consequently, the following dimensions are in use: H [m H_2O], q [m/day], Q [m/day], c_1 [(m H_2O)/(m/day)], c_2[(m H_2O)/(m/day)^n].

Table 1

Coefficients c_1, c_2, n of Eq. (2) measured by Di Bernardo [9], [10] for a pilot and a full scale plants[a]

Scale of the plant	Number of filters	c_1 [(m H_2O)/(m/day)]	c_2 [(m H_2O)/(m/day)^n]	n
Pilot plant	4	0.00145	0.00050	1.22
Full scale plant	4	0.00150	0.0000026	1.90

[a]The coefficients c_1, c_2 were predicted for q_i referred to 1 m^2 (hydraulic load, [$m^3/(m^2\cdot day)$] simplified to [(m/day)]).

The curves $q_i(H)$ in Fig. 4 fit perfectly with the horizontal straight lines expected here in spite of the fact that they are not received from the rough approximation (4), but from the precise numerical solution [11] to the set of Eqs. (1)–(3). Fifty different values of H were chosen, then h_0, c_2 calculated according to the points 1, 2 of the theorem 2 and finally q_i computed from the system of Eqs. (1)–(3). The set of Eqs. (1)–(3) was solved by the most precise of the methods described elsewhere [11]. In this method, each value of q_i is bounded both from the upper and the lower side but these limits are so close to each other that they are denoted by single lines in Fig. 4.

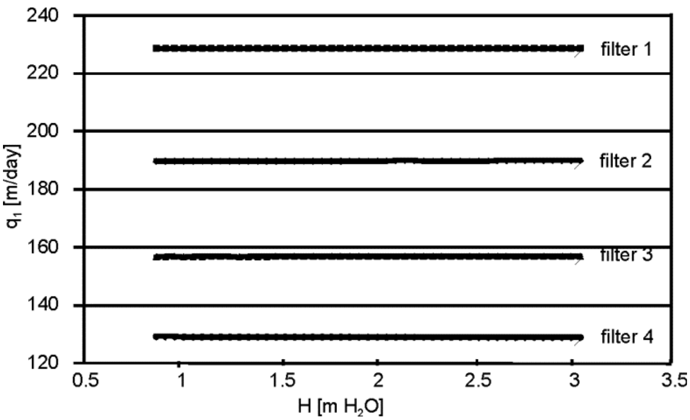

Fig. 4. Distribution of the flow rate vs. H for a family of solutions calculated according to property 1 for Q = 704 m/day, q_1/q_{avr} = 1.3, and for the same data c_1, n as for the pilot plant experiments (Table 1 after [9, 10])

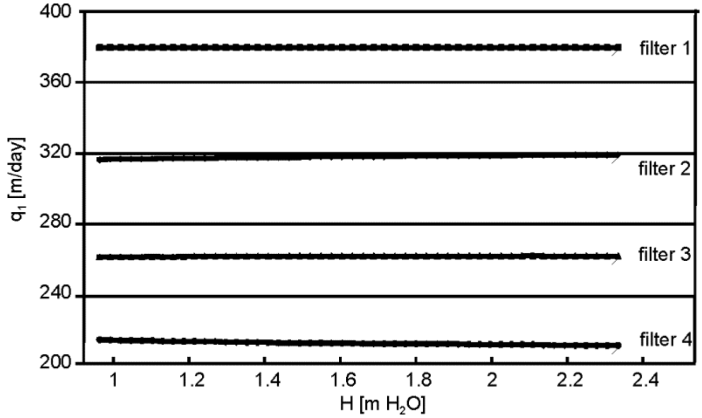

Fig. 5. Distribution of the flow rate vs. H for a family of solutions calculated according to property 1 and for c_1, n the same as given in Table 1 for the full scale water treatment plant and for $q_1/q_{avr} = 1.3$, $Q = 4q_{avr} = 1168$ m/day

The same refers to Fig. 5 but this time the inclinations of the curves $q_i(H)$ from straight horizontal lines are more visible due to a high value of exponent n describing head losses $c_2q_i^n$ created by drainages and orifices.

The next property refers only to the family of solutions to the system of Eqs. (2)–(4) defined by Theorems 1 and 2.

Theorem 3. According to Eqs. (2), (4) and to Theorem 1, the flow rate through a clean filter q_1 and the total capacity of the plant Q remain the same for various coefficients c_2 if simultaneously:

1) the lowest water table level $H - h_0$ varies linearly with c_2 according to Eq. (2):

$$H - h_0 = c_1q_1 + c_2\left(q_1\right)^n$$

2) the highest water table level H before a backwash depends proportionally on c_2;

$$H = \frac{c_1q_1 + c_2(q_1)^n}{1 - \dfrac{h_0}{H}} \tag{5}$$

3) the water level increase between backwashes h_0 fulfils a linear dependence:

$$h_o = \frac{\left(c_1q_1 + c_2\left(q_1\right)^n\right)\dfrac{h_0}{H}}{1 - \dfrac{h_0}{H}} \tag{6}$$

Theorem 3 is illustrated in Figs. 6–9 computed as previously from an accurate solution to Eqs. (1)–(3) for c_1, z, n, Q listed in Table 1 as predicted for a full scale plant and for $q_1/q_{avr} = 1.3$.

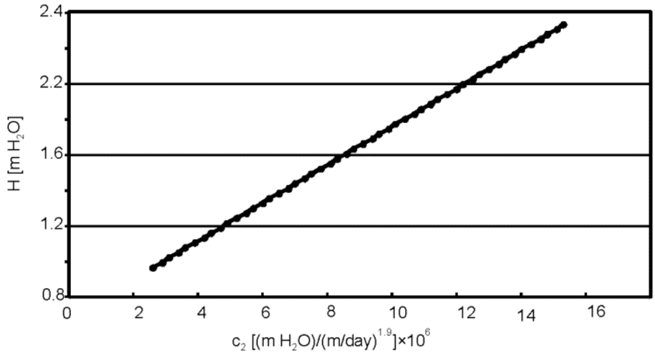

Fig. 6. Total head loss H as a function of c_2 for data presented in Fig. 5

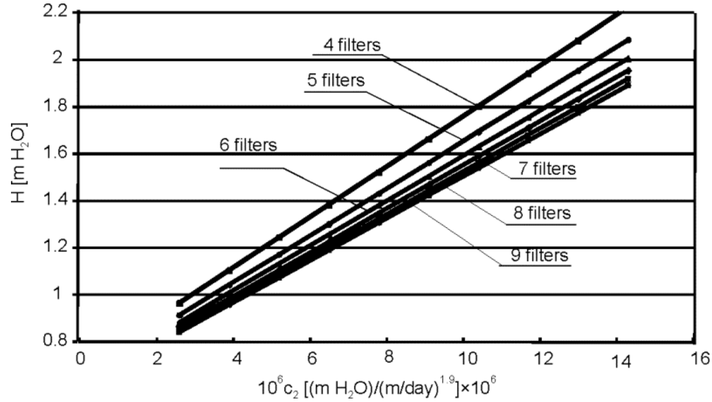

Fig. 7. Head loss H as a function of c_2 (for c_1, n as for the full scale treatment plant reported in Table 1) in comparison with the head loss computed for more filters operating with the same $q_{avr} = Q/z = 292$ m/day, $q_1/q_{avr} = 1.3$

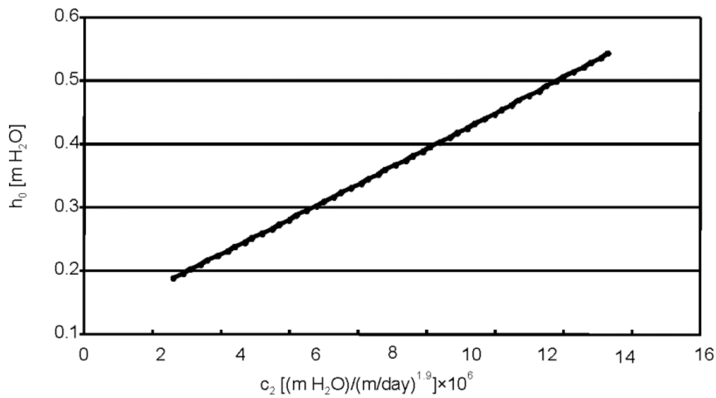

Fig. 8. The highest water table fluctuation h_0 over the filters, computed as a function of c_2 for the same data as the calculations presented in Fig. 5

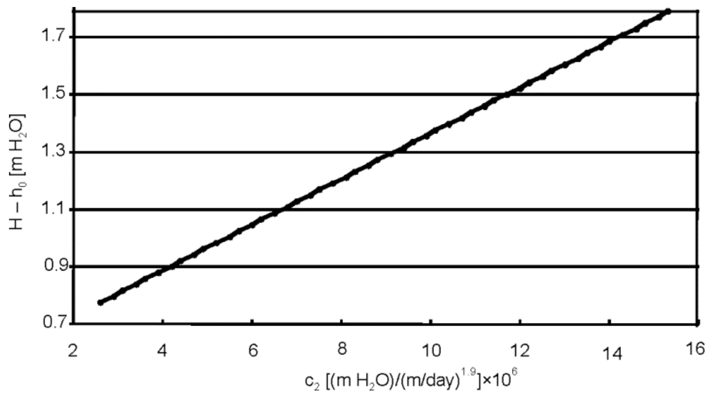

Fig. 9. Head loss $H - h_0$ calculated as a function of c_2 for the same data as in Figs. 5, 6

The next theorem refers to the resistance of hydraulic porous media c_{1z} to flow just before its backwash. As the total head loss of flow through the plant is equal to H just before each of backwashes, c_{1z} is defined by:

$$H = c_{1z} q_z + c_2 (q_z)^n \tag{7}$$

Equation (7) is introduced for the most clogged filter z operated with the flow rate q_z.

Theorem 4. For a family of solutions to the set of Eqs. (2)–(4) constructed according to the points 1, 2 of Theorem 2 (so characterized by the same values of flow rates $q_1, ..., q_i, ..., q_z$) the hydraulic resistance c_{1z} of the most clogged filter media just before its backwash depends linearly on the constant c_2 being proportional to the turbulent head losses in the drainage and orifices:

$$c_{1z} = \frac{c_1 q_1 + c_2 q_1^n - \left(1 - \frac{h_0}{H}\right) c_2 q_z^n}{q_z \left(1 - \frac{h_0}{H}\right)} \tag{8}$$

The family of solutions to the system of Eqs. (4), (2), (3) calculated according to the instructions given by Theorems 1–3 have almost identical flow distribution $q_1, ..., q_i ..., q_z$ between filters in a bank and a constant ratio h_0/H. Equation (8) results directly from the definition of c_{1z} (7) and from Eqs. (2), (6). Theorem 4 is illustrated in Figs. 10, 11 constructed as previously from fifty accurate solutions to the set of Eqs. (1)–(3). The data for these computations are the same as those listed in Table 1, with $q_1/q_{avr} = 1.3$ in this instance. It may be seen that the resistance c_{1z} is almost a linear function of c_2. However, the ratio of c_{1z}/H as a function of c_2 shows some unlinearity as presented in Fig. 12, constructed from the solutions to the system of Eqs. (1)–(3) for 50 different values of c_2.

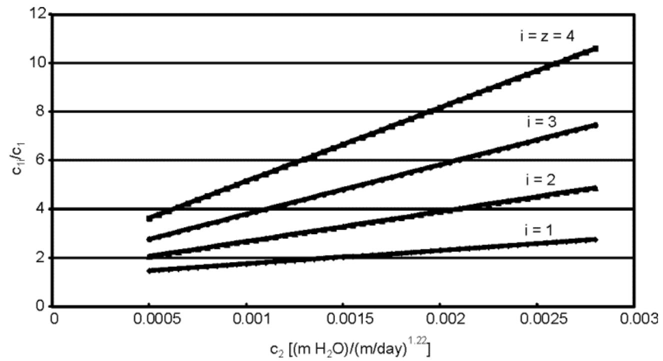

Fig. 10. Relative resistance c_{1i}/c_1 of four filters before backwashing calculated as a function of c_2 for Q = 704 m/day, q_1/q_{avr} = 1.3, and for c_1, n the same as for the pilot plant (see Table 1) ; the most clogged filter 4

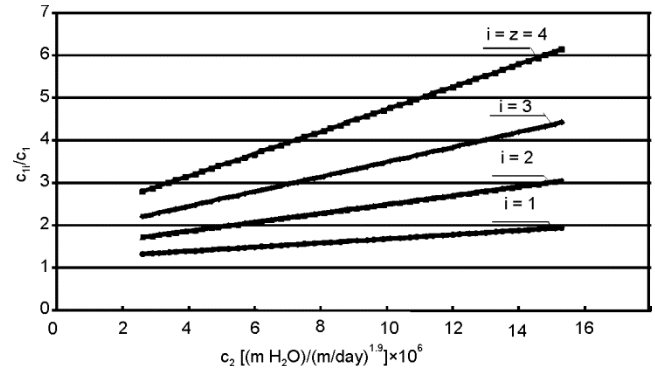

Fig. 11. Resistance of four filters as a function of c_2 for the same n, c_1 as for the full scale plant reported in Table 1, and for q_1/q_{avr} = 1.3, $Q = 4q_{avr}$ = 1168 m/day

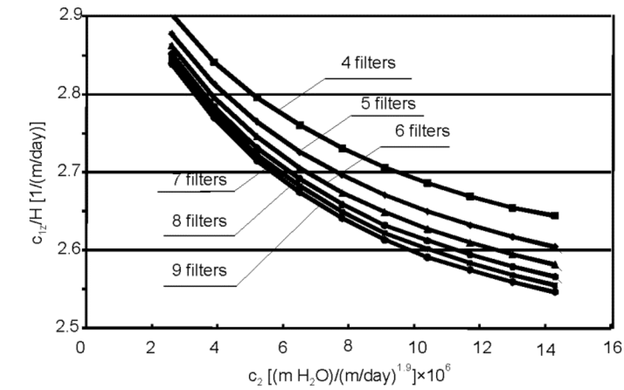

Fig. 12. Some nonlinearity of the ratio c_{1z}/H as a function of c_2 according to computations based on a precise numerical solution to the system of Eqs. (1)–(3) and subject to the same Q, q, c_1, n as for Figs. 5–9, 11

6. OPTIMISATION PROBLEM

It is assumed that the highest resistance of porous media to flow corresponds to the largest volume of water passing through a filter. This usually reasonable assumption is not always valid as the depth bed filtration theory gives a complex picture of the head loss increase with time and filtration velocity impacts a pattern of deposit distribution in depth. However, in most optimisation tasks it is not possible to avoid some simplifications. It is believed that the substitution of a problem on the longest filter run by the problem on the highest media resistance to flow c_{1z} at the end of this run is acceptable in many technical applications. Concluding the objective function is defined by the equation

$$\text{Max} c_{1z} = \frac{H - c_2 q_z^n}{q_z} \tag{9}$$

The constraints include the set of Eqs. (4), (2), (3) and some limits imposed to the total available head loss of flow H and the ratio of flow rate q_1 to the average flow rate q_{avr}:

$$H \le H_{\max} \tag{10}$$

$$\frac{q_1}{q_{\text{avr}}} \le D \tag{11}$$

where: $H_{\max}$ – the highest available head loss of flow through a plant limited by its construction, D – limit imposed on the ratio q_1/q_{avr} because of required filtrate quality.

The formulated optimisation problem will be solved in two steps. First it will be shown that for all families of solutions to the set of Eqs. (4), (2), (3) characterized by flow rate distribution $q_1, ..., q_i, ..., q_z$ for which $q_1/q_{\text{avr}} = D$ the highest hydraulic resistance to flow of the most clogged filter c_{1z}, just before its backwash, refers to the highest coefficient c_2. For this value of c_2 the total head loss of flow through the plant H reaches $H_{\max}$. In the latter step, it will be proved that any solution being outside of these families of solutions for which $q_1/q_{\text{avr}} = D$ refers to a lower value of c_{1z}, than received simultaneously for $q_1/q_{\text{avr}} = D$ and for $H = H_{\max}$.

7. SOLUTIONS

The solution to the optimisation problem inside of the family of fixed values of $q_1, ..., q_i, ..., q_z$, for which $q_1/q_{\text{avr}} = D$ results directly from theorems 1–4. As the parameters c_2, H, h_0 have to be adjusted through Eqs. (2), (5), (6) resulting in the required Q, q_1/q_{avr}, the same h_0/H and q_z, so the resistance of the filter media z in Eq. (8) linearly depends on c_2 and is the highest for the value of c_2 for which $H = H_{\max}$. This might be intuitively expected.

The family of solutions defined by the points 1–3 fulfilling Theorem 3 refers to the highest acceptable value of q_1. There are also solutions to the set of Eqs. (4), (2), (3) fulfilling constrains (10), (11) but not belonging to this family. These solutions satisfy required plant capacity Q for some lower values of the largest flow rate q_1^*, and for different flow rate distributions $q_1^*, ..., q_i^*, ..., q_z^*$ among the filters. To make this text clear, the same notations are used as previously to describe h_0, q_1, ..., q_i, ..., q_z, c_2, c_{1z} when they refer to the solutions of the set of Eqs. (4), (2), (3) being a subject to the restriction $q_1/q_{\text{avr}} < D$, and not belonging to the family of fixed q_1, ..., q_i, ..., q_z, for which $q_1/q_{\text{avr}} = D$. However, in this case they are denoted additionally by an asterisk, thus $q_1^* < q_1$. As the filter media is known in advance and all comparisons between the results of calculations will be done for the same arbitrarily chosen head loss H before a subsequent backwash in a plant, there is no reason to use the asterisk * to distinguish c_1 and H for separate families of solutions.

Let us take into account two different solutions to the system of Eqs. (4), (2), (3), both satisfying the same plant capacity for the same available head loss H. The second of these solutions $q_1^*, ..., q_i^*, ..., q_z^*, c_2^*, h_0^*$ refers to $1 < q_1/q_{\text{avr}} < D$. At the end of a filter run, the resistance of the most clogged porous media may be described by the equation:

$$c_{1z}^* = \frac{H - c_2^*(q_z^*)^n}{q_z^*}, \qquad c_{1z} = \frac{H - c_2 q_z^n}{q_z} \tag{7*}$$

We shall investigate the relations between c_2, q_z, and c_2^*, q_z^* in advance to confront c_{1z} with c_{1z}^*. For further discussion it is useful to have a closer look at the relations between h_0, $H - h_0$ and $h_0^*, H - h_0^*$. According to approximation (4) both flow rate distributions q_1, ...,q_i, ...,q_z and $q_1^*, ..., q_i^*, ..., q_z^*$ consist of elements of geometric progression giving an identical plant capacity $Q = q_1 + ... + q_i + ... + q_z = q_1^* + ... + q_i^* + ... + q_z^*$. Bearing in mind that by definition the flow rate q_1^* is lower than q_1 obviously q_z^* must be higher than q_z. Moreover from Eq. (4) $h_0^*, < h_0$. As both solutions (with and without the asterisk) refer to the same available head loss H, thus $H - h_0^* > H - h_0$, which means that the turbulent head loss coefficients c_2 and c_2^* fulfil the inequality $c_1 q_1^* + c_2^*\left(q_1^*\right)^n > c_1 q_1 + c_2\left(q_1\right)^n$. Keeping in mind that from the definition of q_1 any value of q_1^* is lower than q_1, it is obvious that $c_2^* > c_2$. All the necessary items of information on the relations between c_2^*, c_2 and q_z, q_z^* have been collected $(c_2^* > c_2, q_z^* > q_z)$, and finally the expectation that c_{1z}^* is lower than c_{1z} may be confirmed by discussing the equation:

$$c_{1z} = \frac{H}{q_z} - c_2(q_z)^{n-1}, \qquad c_{1z}^* = \frac{H}{q_z^*} - c_2^*(q_z^*)^{n-1} \tag{7**}$$

Concluding, the highest hydraulic resistance of filter media before backwashing should be sought among the family of solutions q_1, ... q_i, ... q_z, c_2, h_0, subject to $q_1/q_{avr} = D$. Finally, the highest value of c_{1z} (and hopefully the longest filter run) refers to $H = H_{max}$ and to $q_1/q_{avr} = D$.

8. PRACTICAL ASPECTS

A possible application of equations and theorems discussed here is now presented in an optimisation example calculated for a four-filter plant. The conditions assumed are those of di Bernardo [9, 10] for a full scale plant (see Table 1): According to the theorems developed in the previous paragraphs, the highest resistance of the media before a backwash occurs if simultaneously the ratio of q_1/q_{avr} and H reach their maximum values.

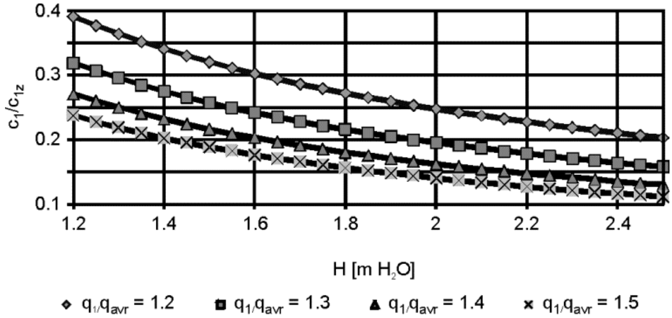

Fig. 13. The ratio of c_1/c_{1z} as a function of H and q_1/q_{avr} for an optimisation example

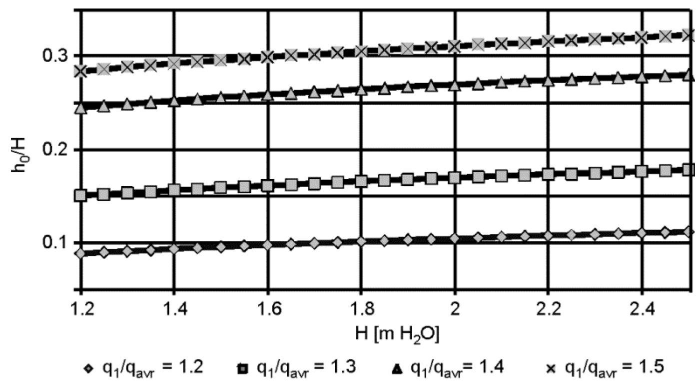

Fig. 14. Values of h_0/H as a function of q_1/q_{avr}
and to some extent also a function of H (data the same as for Fig. 11)

Usually q_1/q_{avr} is limited by technological reasons and H mainly by the construction of a plant. This simple optimisation rule was tested many times in numerical calculations. Examples of such tests are presented in Figs. 13 and 14. Each of lines in these figures was constructed from 50 points obtained from an accurate numerical solution to the set of Eqs. (1)–(3). A full system of equations was used again to verify the optimisation approach developed based on the approximated model described by the set of Eqs. (4), (2), (3). Coefficients c_1, n and the plant capacity are the same as in di Bernardo's [10] full scale experiments. Values h_0, c_2 were adjusted in such a way that the same Q and q_1/q_{avr} were obtained for different values of H varying from 1.2 to 2.5 m. From Figure 13 it can be seen that the resistance of the filter media z increases as q_1/q_{avr} increases (for the same H) and behaves analogously if H increases for the same q_1/q_{avr} . The value of h_0/H remains almost constant for all computations referring to the same q_1/q_{avr} (Fig. 14).

9. CONSTANT VERSUS VARIABLE RATE FILTRATION

To evaluate the advantage resulting from using the VDR control system, as opposed to the constant flow rate system, two filter plants consisted of z = 4 identical filter media, producing the same amount of filtrate, and operated with the same total head loss H were compared. The ratio of c_{1z} (CRF)/c_{1z}(VDRF) was calculated and shown in Fig. 15.

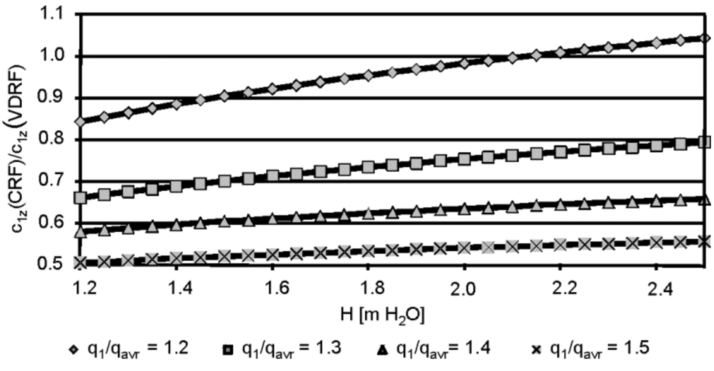

Fig. 15. Ratio of the filter media resistances before a backwash c_{1z} (CRF)/c_{1z}(VDRF), representative of Constant rate and variable declining rate filter operation, respectively

Here c_{1z}(CRF) presents the hydraulic resistance of the constant rate filter (CRF) media z just before the backwash and c_{1z}(VDRF) resistance of the variable declining rate filter (VDRF) media z. The resistance c_{1z}(CRF) was calculated as equal to (H – 0.2 m)/q_{avr}, on the assumption that the head loss of flow through the filter drainage and an open flow rate controller (at the end of the filter run) is equal to 0.2 m H_2O.

For $q_1/q_{avr} = 1.1$ the resistance of the clogged filter media is usually higher in a CFR system than it is in a VDR one. Therefore it is possible that no profit would result from VDR control system and these results are not included in Fig. 15. For $q_1/q_{avr} = 1.2$ VDR control system results in higher hydraulic resistance c_{1z} of filter media z for $H < 2.0$ m. Higher the ratio q_1/q_{avr}, lower the ratio c_{1z}(CRF)/c_{1z}(VDRF) is and more profitable is the VDR operation system. However, the ratio q_1/q_{avr} is to be restricted because of the filtrate quality. A limit of 1.3 or more commonly 1.5 is imposed on this ratio on a rule of thumb but pilot plant experiments should be used to verify these limits for a given case. In such pilot experiments particles counting is much more reliable than turbidity measuring.

10. THE MOSTLY CLOGGED FILTER

Till now only the maximum ratio of q_1/q_{avr} was concerned as a significant factor of the filtrate quality to be determined experimentally. However, the turbidity of filtrate produced by the dirtiest filter may be of crucial concern. The product $h_{media}q$ is sometimes recognized as a representative parameter for filtrate quality relating to given raw water, pretreatment, and porous media. By h_{media} head loss created by filter media is created. The flow rates q_z (four filters) obtained according to precise solution to the set of Eqs. (1)–(3) are denoted by circles in Fig. 16 in comparison with a monogram q_z/q_{avr} constructed as a function of h_0/H from Eq. (12):

$$\frac{q_z}{q_{avr}} = \frac{z\left(1-\frac{h_0}{H}\right)^{(z-1)}}{\frac{h_0}{H}\left(1-\left(1-\frac{h_0}{H}\right)^{z}\right)} \tag{12}$$

resulting directly from approximation (4) and from the formula for a sum of geometrical progression elements q_1 ..., q_i ..., q_z. The general monogram can be applied to any VDR water filtration plant but it is based on rough approximation (4), so the points denoted by circles do not lay exactly on the line developed for a four filter plant. Each of these points represents a value of q_z/q_{avr} computed from the set of Eqs. (1)–(3) for tests in which q_1/q_{avr} = 1.1, 1.2, 1.3, 1.4 and 1.5. For these particular tests q_z versus h_0/H are presented in Fig. 16. If for instance a total available head loss H in this plant is equal to 2.2 m, $h_0 = 0.61$ m, and $q_1 = 438$ m/day $= 1.5q_{avr}$ was chosen experimentally it is advisable to verify also the quality of filtrate collected from the mostly clogged filter ($c_{1z} = c_1/0.12$, see Fig. 13) operated with $q_z = 0.58q_{avr} = 0.58q_1/1.5$ $= 169$ m/day (Fig. 16). If the filtrate quality is poor, it is a question of economy whether it is reasonable to improve pretreatment or whether a lower H, rather than the maximum available value, should be considered. In general, optimising of chemical treatment should be attempted first as it has usually a tremendous impact on both filtration quality and length of a filter run.

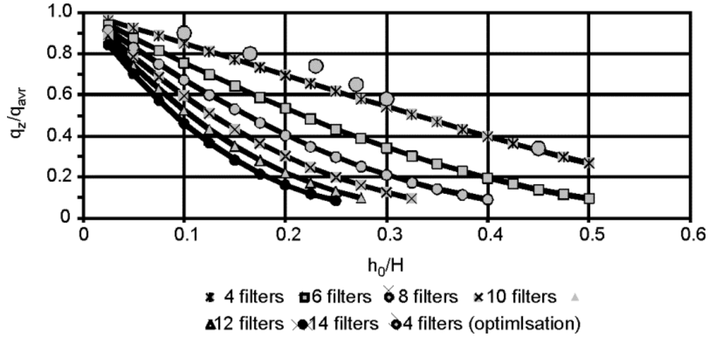

Fig. 16. Monogram for primary values q_z in comparison with some points obtained from an optimisation example

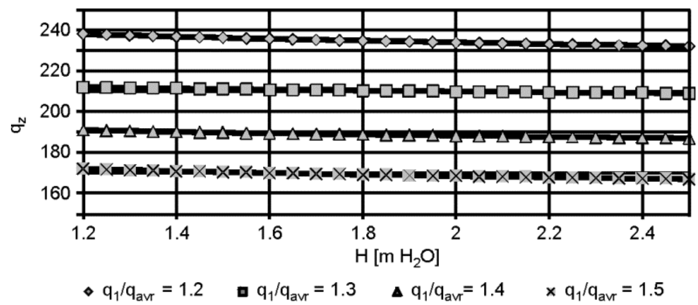

Fig. 17 Almost constant values of q_z versus H calculated for an optimisation example

11. FINAL REMARKS

The theorems presented in the paper and the optimisation approach to operation of a VDR filter plant were developed from the set of Eqs. (4), (2), (3) but verified by numerical solutions to the set of Eqs. (1)–(3), which does not include a simplified Eq. (4). The data from a pilot and a technical scale treatment plants reported by di Bernardo [9, 10] were used for verification of the findings. However, this verification is still limited and partially numerical, so a pilot plant was constructed at the Cracow University of Technology [24] to carry out empirical tests. The results of experiments confirming the theory presented here and advantages of application of the optimisation approach to both a pilot and a full-scale treatment plants will be described in the next paper.

12. CONCLUSIONS

Approximation (4) has been found to be useful in deducing some approximate linear relationships between the parameters of a VDRF hydraulic control system. Based on this approximation, it has been shown that the highest resistance of filter *z* media before backwashing is related to the highest available head loss of flow through a plant and to the turbulent resistance of orifices adjusted to obtain as high a flow rate through a clean filter as technologically possible. The deduced properties of the solution to the system of equations governing VDR filter plants have been verified in comparison with numerical solutions, which in turn have been compared with literature experimental data.

It is expected that the longest duration of filter runs is related to the highest resistance of the filter media just before the backwash. In conclusion, the theoretical considerations presented here led to a more efficient filter plant operation method based on an increase of both the water table level above filter media and the ratio of maximum to average flow rates to the highest acceptable values.

Perhaps the best solution to optimisation problems is a pilot plant experiment taking into consideration some of the linear relations developed in the present paper (Theorems 1–4). Such a pilot plant and calculations would have to take into account any possible changes in the water temperature in summer and winter [25].

SYMBOLS

c_1	–	proportional coefficient characterizing the resistance of a clean medium
c_2	–	coefficient of turbulent head losses
c_{1z}	–	resistance of filter media *z* before a backwash
c_{1z}(CRF)	–	resistance of filter media *z* for constant flow rate control system
D	–	constant value
H	–	total head loss of flow through the plant just before a backwash
h_{media}	–	head loss of flow through filter media just before its backwash
h_0	–	the height of water surface increase between backwashes
i	–	number of a filter in a bank
n	–	exponent of turbulent head loss
Q	–	total flow rate through the system
q_i	–	flow rate through a filter i
q_{avr}	–	mean value of flow rate through a filter $q_{\mathrm{avr}} = Q/z$
z	–	number of the filters in a bank

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