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# MATRIX METHOD FOR ESTIMATING THE RISK OF FAILURE IN THE COLLECTIVE WATER SUPPLY SYSTEM USING FUZZY LOGIC

Collective water supply system (CWSS) belongs to the so called critical infrastructure with a priority importance for urban agglomerations. The draft of European standard *Security of drinking water supply. Guidelines for risk and crisis management* defines the concept of risk in CWSS and principles of management and protection in crisis situations. In the paper, a method for estimating the risk of failure in CWSS, based on the assumptions of classical matrix methods while information from the system operation is inaccurate, was proposed. The proposed method is based on fuzzy logic, which allows one to incorporate incomplete data and develop a risk evaluation system.

## 1. INTRODUCTION

Collective water supply systems (CWSS) are the foundation of existence of urban agglomerations. As one of the elements of an expanded urban infrastructure the system is exposed to the risk of an external nature (floods, droughts, blackout, incidental pollution of water sources, earthworks, ground temperature changes as well as acts of vandalism and even terrorism) and internal nature (technical failure resulting from processes of aging, fatigue, or hydraulic conditions, technical defects of materials and failures being a result of poor workmanship or wrong operation) [1, 2]. As a result of mentioned above incidents CWSS or one of its subsystems (intake, pumping and storage, distribution) may fail, the consequences are born by water consumers and water supply companies [3, 4]. A measure of the risk of failure in CWSS is a function of the probability of the occurrence of undesirable events and possible losses resulting from such events. In the analyses of risk in CWSS, we often find the problem of the ambiguity of the available operation data or data from the experts. A theory that can be

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used in such a case is the theory of fuzzy sets [5–7]. The concept of fuzzy sets was introduced in 1965 by Zadeh [7] from the University of Berkeley (California), who based on the work of Polish mathematician (creator of multi-valued logic) Jan Łukasiewicz. Fuzzy logic can be viewed as a multi-valued logic. Unlike in the classical set theory, the limit of the fuzzy set is not precisely determined but a gradual transition occurs from non-membership of elements in a set, through their partial membership, to full membership. This gradual transition is described by the so called membership function $\mu_A$. Fuzzy sets can be used to describe various linguistic concepts related to risk analysis (small, medium, large, very large). Membership function $\mu_A$ assigns a value from the interval [0, 1], $\mu_A: X \to [0, 1]$ to each element $x$ of the universe $X$. Most commonly used shapes for membership functions are Gaussian, triangular or trapezoidal [5–7].

## 2. MATERIALS AND METHODS

### 2.1. FUZZY MODEL OF THE RISK OF FAILURE OF WATER NETWORK

Risk is a measure of the probability and severity of the adverse effect [8, 9]. For the CWSS, the measure of risk ($r$) is defined as:

$$r = f(P, C) = \sum_{S} PC \tag{1}$$

where: $S$ – a series of the successive undesirable events (failures), $P$ – the probability (likelihood) of $S$ or a single failure (a point value, depending on the frequency of failure), $C$ – a point value of losses caused by $S$ or a single failure. Depending on the frequency of a given failure, the point weights for the parameter $P$ are presented in Table 1 [9, 11].

Table 1

Criteria for a descriptive point scale for the parameter $P_i$ ($i$ = 1, 2, 3)

| $P_i$ | Probability of failure |
|---|---|
| 1 | low probability, once in (2–10) years and less often |
| 2 | medium probability, once in (0.5–2) years |
| 3 | high probability, once in (6–12 ) months and more often |

The criteria and the point weights for the assumed descriptive point scale for the parameter of losses $C_j$ are presented in Table 2 [11, 12].

Table 2

Criteria for a descriptive point scale for the parameter $C_j$, ($j = 1, 2, 3$)

| Point weight $C_j$ | Description |
|---|---|
| 1 | small losses: perceptible organoleptic changes in water, isolated consumer complaints, financial losses up to $5\times10^3$ € |
| 2 | medium losses: considerable organoleptic problems (odour, changed colour and turbidity), consumer health problems, numerous complaints, information in local public media, financial losses up to $10^5$ € |
| 3 | large losses: endangered people require hospitalisation, professional rescue teams involved, serious toxic effects in test organisms, information in nationwide media, financial losses > $10^5$ € |

Based on Eqs. (2) and Tables 1, 2 two-parametric risk matrix was formulated:

$$r_{ij} = \begin{vmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{vmatrix}$$

According to the basic matrix for risk assessment given above we can analyse various undesirable events, assuming the following scale of risk: tolerable risk ($r_T$), controlled risk ($r_C$), and unacceptable risk ($r_U$).

For risk analysis of water mains failure the membership function class type $t$ (a triangular function, Eq. (2)), the membership function class type $\gamma$ (Eq. (3)) and the membership function class type $L$ (Eq. (4)), were proposed [13, 14].

$$\mu_A(x,a,b,c) = \begin{cases} 0 & \text{for } x \le a \\ \dfrac{x-a}{b-a} & \text{for } a < x \le b \\ \dfrac{c-x}{c-b} & \text{for } b < x \le c \\ 0 & \text{for } \quad x > c \end{cases} \tag{2}$$

$$\mu_A(x,a,b) = \begin{cases} 0 & \text{for } x \le a \\ \dfrac{x-a}{b-a} & \text{for } a < x \le b \\ 1 & \text{for } x > b \end{cases} \tag{3}$$

$$\mu_A(x,a,b) = \begin{cases} 1 & \text{for } x \le a \\ \dfrac{b-x}{b-a} & \text{for } a < x \le b \\ 0 & \text{for } x > b \end{cases} \tag{4}$$

Table 3 shows the linguistic characterization, type and parameters of the membership function.

Table 3

Linguistic characterization, type and parameters of the membership function for $P$ parameter, $\overline{P} = \{A_1, A_2, A_3\}$

| $A_i$ | Linguistic characterization | Type of the membership function | Parameters of the membership function | | |
|---|---|---|---|---|---|
| | | | $a$ | $b$ | $c$ |
| $A_1$ | low probability | type $L$ (Eq. (4)) | – | 0.125 | 0.5 |
| $A_2$ | medium probability | triangular $t$ (Eq. (2)) | 0.125 | 0.50 | 0.75 |
| $A_3$ | high probability | type $\gamma$(Eq. (3)) | 0.5 | 0.75 | – |

Table 4 shows the linguistic characterization, type and parameters of the membership function for the $C$ parameter.

Table 4

Linguistic characterization, type and parameters of the membership function for the $C$ parameter, $\overline{C} = \{B_1, B_2, B_3\}$

| $B_j$ | Linguistic characterization | Type of the membership function | Parameters of the membership function | | |
|---|---|---|---|---|---|
| | | | $a$ | $b$ | $c$ |
| $B_1$ | small | triangular $t$ (Eq. (2)) | 0.0 | 0.0 | 1.5 |
| $B_2$ | medium | | 0.5 | 1.5 | 2.5 |
| $B_3$ | large | | 1.5 | 3.0 | 3.0 |

Figure 1 shows the forms of the membership function for the parameters $P$ and $C$.

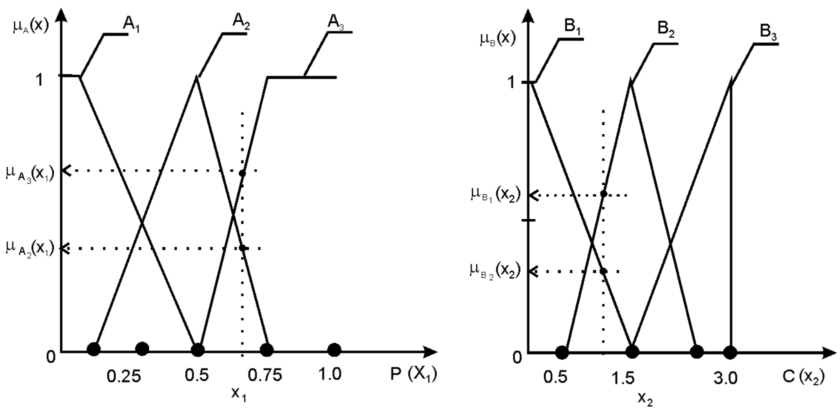


Fig. 1. Form of the membership function for the parameters $P(x_1)$ (left) and $C(x_2)$ (right)

### 2.2. EXPERT SYSTEM USING FUZZY RULES OF INFERENCE

A fuzzy decision model calculates the output value based on the multiple input values. The model does not analyse the exact values of the arguments, only their grade of membership in fuzzy sets, and thus the output value, being a base for the decision making process, is determined. The model consists of four main blocks [13]:

- The fuzzification block, which converts a vector of numbers (the crisp input values) into a vector of the grades of membership (e.g. a singleton method). For each input several fuzzy sets can be attributed. For each crisp value the membership to all sets connected with given input is calculated. For risk analysis, the input values for the model are risk parameters: variable $x_1$ (probability) and variable $x_2$ (losses). For the value of the probability, variable $x_1$ may belong to three defined sets: $A_1$ – low probability, $A_2$ – medium probability, $A_3$ – high probability, for the parameter of losses $C$ variable $x_2$ may belong to the sets: $B_1$ – small, $B_2$ – medium, $B_3$ – large.
- The block of rules – determining the relationship between the inputs and outputs of the expert system (determination of the rules of inference). The base of rules represents the expert knowledge of possible values of variables characterizing the analysed system. The base is a set of fuzzy rules: $\{R_1, R_2, \ldots, R_M\}$ in a *if-then* form.
- The inference block – the determination of a fuzzy conclusion of the expert system. There are several models of inference; the most popular models are Mamdani –Zadeh or Takagi–Sugeno models [5].
- The defuzzification block – the crisp output value is calculated for the inferred fuzzy set. We obtain a specific result, which completes the decision making process. The transformation of fuzzy set into the determined value can be made by various methods [5, 13].

## 3. RESULTS

### 3.1. MODEL OF EXPERT SYSTEM FOR RISK ANALYSIS IN CWSS OF THE TAKAGI–SUGENO TYPE

The basic feature of the Takagi–Sugeno–Kang (TS) inference model is the lack of defuzzification block because the result of the model is not in a fuzzy form. A typical rule in a TS fuzzy model has the form [5–7]:

If $x_1$ is $A_i$ and $x_2$ is $B_j$, then output is $y = r_{ij}$.

The most frequently used function is the first order polynomial:

$$y = p_0 + \sum_{i=1}^{N} p_i x_i \tag{5}$$

where $p_0, p_1, \ldots, p_N$ are numerical weights. For a zero order TS model $p_i = 0$ for $I > 0$.

The output level $y_i$ of each rule is weighted by the firing strength $w_i$ of the rule. The output from the model takes the form:

$$y = \frac{\sum_{i=1}^{3}\sum_{j=1}^{3} w_{ij} r_{ij}}{\sum_{i=1}^{3}\sum_{j=1}^{3} w_{ij}} \tag{6}$$

$$w_{ij} = \mu_{Ai}(x_1) T \mu_{Bj}(x_1) \tag{7}$$

where $T$ is the algebraic product.

For the proposed model of assessing the risk of failure in CWSS a multi-input single-output (MISO) TS model was proposed, Matlab program (fuzzy toolbox) was used and the following assumptions were made: model consists of two inputs (value of the probability $x_1$ and value of the consequences $x_2$). Each variable is defined by fuzzy sets: $\{A_1, A_2, A_3\}$ for variable $x_1$ (Table 3, Fig. 1) and $\{B_1, B_2, B_3\}$ for variable $x_2$ (Table 4, Fig.1). The output from the model is a positive real number $r_{ij}$ describing the risk of failure in CWSS. It is defined by three possible values: $r = \{r_T, r_C, r_U\}$.

Table 5 presents a description of the base of rules for the proposed model, developed based on literature data [10, 13, 15]. This base of rules was introduced into the base of Matlab Fuzzy Toolbox.

Table 5

Base of rules of the expert system: risk analysis in CWSS

| Number of the rule | Description |
|---|---|
| R1 | If $x_1$ is $A_1$ and $x_2$ is $B_1$ then $y = r_{11}$ |
| R2 | If $x_1$ is $A_1$ and $x_2$ is $B_2$ then $y = r_{12}$ |
| R3 | If $x_1$ is $A_1$ and $x_2$ is $B_3$ then $y = r_{13}$ |
| R4 | If $x_1$ is $A_2$ and $x_2$ is $B_1$ then $y = r_{21}$ |
| R5 | If $x_1$ is $A_2$ and $x_2$ is $B_2$ then $y = r_{22}$ |
| R6 | If $x_1$ is $A_2$ and $x_2$ is $B_3$ then $y = r_{23}$ |
| R7 | If $x_1$ is $A_3$ and $x_2$ is $B_1$ then $y = r_{31}$ |
| R8 | If $x_1$ is $A_3$ and $x_2$ is $B_2$ then $y = r_{32}$ |
| R9 | If $x_1$ is $A_3$ and $x_2$ is $B_3$ then $y = r_{33}$ |

The particular risk sets are:
- the tolerable risk set: $r_T = \{r_{11}, r_{12}, r_{21}\}$,
- the controlled risk $r_C = \{r_{13}, r_{22}, r_{31,} r_{23}\}$,
- the unacceptable risk $r_U = \{r_{32}, r_{33}\}$.

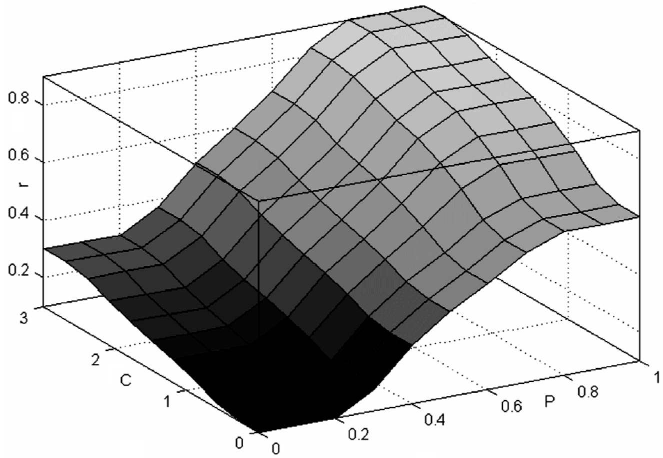


Fig. 2. The risk surface

The risk graph generated from the Matlab Fuzzy Toolbox program is shown in Fig. 2.

## 4. CONCLUSIONS

In the proposed MISO TS model for assessing the risk of failure in CWSS, each of the input variables ($x_1 = P$ and $x_2 = C$) is defined by three linguistic values, respectively:

- for $x_1$ (Table 1): low probability ($A_1$), medium probability ($A_2$), and high probability ($A_3$);
- for $x_2$ (Table 2): small ($B_1$), medium ($B_2$), large ($B_3$).

Inference in the proposed expert model is based on the Takagi–Sugeno–Kang procedure (the risk of failure in CWSS). The concept of fuzzy inference presented in the paper may be considered as a tool to support decisions made in the process of managing risk of failures in CWSS. Based on the determined risk, the system operator makes decisions concerning the operation or modernization (repair) of the system or its individual components, in particular: if as a result of risk analysis the model generates a value of tolerable risk then the system operator decides to allow the system to operate. If as a result of risk analysis the model generates a value of controlled risk then the system operator decides to allow the system to operate but under the condition that modernization or repair will be undertaken. If as a result of risk analysis the model generates a value of unacceptable risk then the system operator decides not to allow the system to operate and initiates an immediate action to reduce risk. The proposed model can be used both for a comprehensive risk analysis in CWSS as well as for the partial analysis of its individual elements. For example, in the process of modernization, the components with the highest risk of failure will be modernized first.

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