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## NUMERICAL EXPERIMENTS INTO OPTIMISATION OF VDR FILTERS

A comprehensive model of the filtration process, that includes the effect of deposition on head loss and removal efficiency, has been combined with a hydraulic model of a Variable Declining Rate (VDR) plant. This model has been used to study the behaviour of VDR filters. These results have been compared to the operational rules deduced from an earlier simplified analysis of the hydraulics of VDR filter plants. The new model has confirmed the validity of these rules, in particular that the filter run is longest if simultaneously the head loss and the ratio of maximum to average flow rates are equal to the highest acceptable values.

### NOTATION

$c_1$  – a constant parameter describing the hydraulic resistance of clean porous media,  
 $c_2$  – the coefficient of friction created by drainage and an orifice,  
 $h_0$  – the height of water table fluctuation between backwashes,  
 $h_r$  – the head losses in drainages and orifices,  
 $H$  – a total head loss of flow through a filter just before a backwash,  
 $n$  – the coefficient,  
 $q$  – the flow rate through an orifice,  
 $q_i$  – the flow rate through  $i$ -th filter,  
 $z$  – the number of filters in a bank.

### 1. INTRODUCTION

Variable Declining Rate (VDR) operation of filter plant is known to be an economic alternative to the more common constant rate operation. These filters are equipped with

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orifices or partially open butterfly valves instead of the flow-rate controllers required for constant rate. The hydraulic operation of the plant is governed by the interaction between the turbulent head losses in the orifices and the laminar head losses in the filter media. For clean filters the turbulent head losses are dominant and restrict the flow rate through recently backwashed filters, for clogged filters the head losses in the media are much more important.

ARLBOLEDA et al. [1] reported that worldwide over 500 filter plants were operated under VDR control system, 300 in Brazil alone [2]. The high cost of the flow rate controllers needed for constant rate operation is one of the reasons for the popularity of VDR operation. However, there are advantages in terms of efficiency, such as the possibility of extending filter runs [3], [4] or increasing plant capacity [4]. Variable Declining Rate system of operation requires inflow of raw water to be located below the lowest water table level above filters and negligible head losses of flow through the piping in comparison with those created by flow through filters (media and orifices). These two requirements ensure that the water table level is the same above all filters throughout operation. This level takes the highest value just prior to a backwash in a plant, and is denoted here by  $H$ . During a backwash one of filters is disconnected so the other filters accept more water, which results in increased flow rates through these filters, head loss and water table position [5]. After the freshly backwashed filter is brought back into service the outflow of the plant increases causing decrease in water level. This decrease is denoted here by  $h_0$ . Flow rates through individual filters change between subsequent backwashes in a plant, but these changes are small compared to the change in head loss after backwashes, and it is reasonable to treat the flow rate as being constant between backwashes. This important finding [3] is useful in constructing simple theoretical models of VDR filters used for many industrial applications. One of these models developed by DI BERNARDO [6] was used by DĄBROWSKI [7], [8], [9] for predicting the parameters of a filter plant operation to ensure the maximum length of filters run. These models make a number of other assumptions, particularly regarding the kinetics of removal within the filter. Therefore numerical simulations of VDR filter plant operation have been carried out using more sophisticated models of the removal processes in a filter, and these simulations are used to validate the rules of VDR filter plant operation.

## 2. PURPOSE OF RESEARCH

Ideally, the parameters of filter plant operation should be chosen with the aid of an rational optimisation approach. Based on the work of DI BERNARDO [6], [10], DĄBROWSKI [7] found that flow rates through different filters could be approximated by a geometric progression. Using this result, he solved an optimisation

problem based on the parameters of filter operation which ensures the longest possible filter run. He assumed that the longest run, or more strictly the largest volume of filtrate produced between backwashes, corresponded to the highest resistance of clogged filter media. Thus the objective function was the hydraulic resistance of filter media just before a backwash, and this was maximised subject to constraints imposed on:

1. The total head loss  $H$  of flow through a filter just before a backwash.
2. The ratio of the maximum  $q_{\max}$  to the average  $q_{\text{avr}}$  flow rate through a filter unit.

The first constraint is determined by the maximum allowable water level, and the second is necessary in order to ensure that the filtrate concentration is not too high.

Assuming that the flow rates through the filter units formed a geometric progression enabled several properties of VDR filters to be deduced [7]. However, as the precision of this approximation is unknown and work on VDR filters does not take full account of changes in removal efficiency within the filters, verification of the optimisation procedure is needed. Experimental verification of the optimisation procedure has been already successfully done [11], [12] for several hydraulic filtration parameters. Now a computer simulation approach is adopted for testing this procedure. MACKIE and ZHAO [13] developed a comprehensive model of removal in deep-bed filtration: this model takes account of the non-homogeneity of both the suspension and the filter media. This removal model was incorporated in a hydraulic model of a VDR filter plant. The first time the Unit Bed Element approach has been applied to modelling VDR filter plants. It does not include the approximations assumed previously by DĄBROWSKI [7] who used a simplified DI BERNARDO model [6]. The work of DĄBROWSKI [7] produced the following operation rules:

- Flow rates through all the filters can be controlled by simultaneously setting the flow rate through the clean filter and controlling the ratio of the water table fluctuations  $h_0$  to the total head loss  $H$ .
- In the case of fixed values of flow rates  $q_i$  through the filters, both  $H$  and  $h_0$  linearly depend on the coefficient  $c_2$ , used in equation (1), for calculation of turbulent head loss of flow through an orifice  $\Delta h_{\text{loss}}$  installed at outflow from each of the filters.
- Maximising both  $H$  and  $q_1/q_{\text{avr}}$  also maximises the filter media resistance to flow immediately prior to backwash.

Each of these statements will be tested by numerical experiments based on the UBE model by MACKIE and ZHAO [13] applied to VDR mode of operation. Equation (1) describes the head loss  $\Delta h_{\text{loss}}$  for flow  $q_i$  through an orifice:

$$\Delta h_{\text{loss}} = c_2 q_i^n, \quad (1)$$

where  $c_2$  and  $n \leq 2$  are empirical coefficients.

### 3. VDR FILTER MODEL

The filtration model used in these studies has been described in detail elsewhere [13], therefore only a brief description will be given here. For a polydisperse suspension the kinetic and continuity equations are:

$$\frac{\partial C_i}{\partial L} = -\lambda_i C_i, \quad i = 1, \dots, n, \quad (2)$$

$$\frac{\partial \sigma_i}{\partial t} + u \cdot \frac{\partial C_i}{\partial L} = 0, \quad i = 1, \dots, n, \quad (3)$$

$C$  is the concentration,  $L$  – the depth,  $\lambda$  – the filter coefficient,  $\sigma$  – the (absolute) specific deposit, and  $u$  – the superficial velocity. The first equation describes the removal, and the second is a simplified [14] mass balance equation. The suspension was assumed to consist of the particles of  $n$  different sizes, and the subscript  $i$  refers to the  $i$ -th size group. The functional form used for the filter coefficient was:

$$\begin{aligned} \lambda &= \lambda_0 + (\lambda_1 - \lambda_0) \left( 1 - \exp\left(-\frac{\sigma^2}{\sigma_1^2}\right) \right), \quad \sigma \leq \sigma_m, \\ \lambda &= \lambda_m (\sigma_u - \sigma), \quad \sigma_m \leq \sigma \leq \sigma_u, \\ \lambda &= 0, \quad \sigma > \sigma_u, \end{aligned} \quad (4)$$

where

$$\lambda_m = \left[ \lambda_0 + (\lambda_1 - \lambda_0) \left( 1 - \exp\left(-\frac{\sigma^2}{\sigma_1^2}\right) \right) \right].$$

$\lambda_0$  is the initial filter coefficient,  $\lambda_m$  is the maximum value of  $\lambda$ , and this value is reached when the specific deposit reaches a value of  $\sigma_m$ . The first part of equation (4) represents a curve that increases initially, then tends asymptotically to the maximum value. Once  $\sigma$  has reached the value  $2\sigma_1$  there is very little change in the value of  $\lambda$ .

Whether or not this stage actually occurs depends upon the values of  $\sigma_m$  and  $\sigma_1$ . When  $\sigma$  reaches  $\sigma_m$  the removal efficiency goes into decline until it finally reaches zero at  $\sigma_u$ . In a polydisperse suspension, it is possible for  $\sigma$  to exceed  $\sigma_u$ , because when the larger particles have stopped being collected, the smaller ones are still being removed. Therefore it is necessary to specify that the filter coefficient is zero for  $\sigma > \sigma_u$  for a given  $i$ -th size group. The values of the parameters in equation (4) will, in general, be different for each particle type. The values of the parameters were calibrated against a set of experimental results under one set of conditions. Simple theo-

retical models were then used to predict how these parameters will change with changing operating conditions such as flow rate, temperature and grain size. The removal model was combined with a simple head loss model. This assumed a quadratic form for the change in head loss gradient with specific deposit. The effect of different grain sizes, temperature, grain size and flow rate were assumed to follow the relationships contained within the Carman–Kozeny equation.

The above filter model was incorporated within a hydraulic model of a VDR filter plant. The head loss through each individual filter and its associated pipe work is assumed to follow the relationship:

$$h = c_1 q_i + c_2 q_i^n . \quad (5)$$

The first part represents the laminar head loss in the filter, with the value of  $c_1$  being determined by the head loss equation used. The second part represents the turbulent head losses, with  $n$  usually being between 1.5 and 2.0. The advantage of this combined model is that it takes account of the effects of the flow rate variations inherent in VDR operation on removal within the filters. It should also be noted that more advanced head loss models are currently being developed and these can be incorporated into the overall model without difficulty.

It was assumed that:

- raw water inflows were located well below the lowest water table above the filters,
- outflow of the filter plant was constructed in a way protecting the media against unsaturated filtration,
- head losses in piping were negligible compared with the head loss of flow through each of the filter.

The numerical model of the VDR filter plant operation including UBE modelling method presented above was applied before to predict the impact of the quality of raw water upon filter operation [15] and to compare the removal efficiency for VDR and CR filters [16], [17].

#### 4. DATA FOR NUMERICAL EXPERIMENTS

The model described above was used to test the VDR filter operation rules developed by DĄBROWSKI [9]. Size fractions of suspended particles used in the computations are reported in table 1 and stratification of the filter grains is given in table 2.

The initial porosity was assumed to be 0.42. The plant consisted of four units. The water suspension density was constant and equal to  $1000 \text{ kg/m}^3$  because of low suspended solids concentration, while the density of suspended particles was assumed to be equal to  $1360 \text{ kg/m}^3$ .

Usually in the literature on VDR filters [6], [7] the flow rate  $q_i$  through a filter  $i$  refers to the flow through one square meter of the filter media, which requires an

adjustment of the coefficient  $c_2$  value in equation (1). The coefficient  $n$  was assumed to be equal to 1.9, which is close to the maximum value 2.0 for fully turbulent flow. In many technical applications when head losses in the drainage are high compared to

Table 1

Size fraction of the suspension solid particles in a raw water

| Group | Size range<br>( $\mu\text{m}$ ) | Representative diameter<br>( $\mu\text{m}$ ) | Volumetric<br>concentration |
|-------|---------------------------------|--|-----------------------------|
| 1     | 0.63–1.26                       | 0.90   | $0.9 \times 10^{-6}$        |
| 2     | 1.26–2.52                       | 1.80   | $1.1 \times 10^{-6}$        |
| 3     | 2.52–5.04                       | 3.60   | $2.5 \times 10^{-6}$        |
| 4     | 5.04–10.09                      | 7.20   | $2.6 \times 10^{-6}$        |

Table 2

Stratification of the grains in the filter

| Depth of media<br>(mm) | Grain diameter<br>(mm) |
|------------------------|------------------------|
| 0                      | 0.467                  |
| 100                    | 0.48                   |
| 600                    | 0.704                  |
| 900                    | 0.768                  |
| 1000                   | 0.832                  |

the ones created by orifice, the exponential coefficient  $n$  is lower. Overall,  $n$  usually lies in the range of  $1.5 \leq n \leq 2.0$ . However, the main purpose of the present study is to verify the properties of VDR filter operation deduced previously [7] from an approximation of flow rates through filters by elements of a geometrical progression. This approximation would be exact if non-turbulent head losses controlled the flow distribution between the filter units and is less precise otherwise. Therefore assuming a high value of the coefficient  $n$  in the computations provides a more severe test. The total head loss  $H$  just before a backwash and the value of  $c_2$  were adjusted in each numerical simulation in order to achieve a flow rate ratio of  $q_1/q_{\text{avr}}$  equal to or lower than 1.3 and the required value of  $h_0$ . The value for  $q_1/q_{\text{avr}}$  is based on US literature for VDR plant operation [3],  $q_1$  denotes the flow rate through a clean filter, and  $q_{\text{avr}}$  – the average flow rate through a plant. The value of  $q_{\text{avr}}$  was specified to be equal to  $8 \text{ m}^3/\text{m}^2\text{h}$  (filtration velocity of  $8 \text{ m/h}$ ), and this determines the total flow through a plant. The patterns of flow rates and water table fluctuations above filters are described elsewhere [1], [2], [3], [18] and are illustrated here in figure 1 by an example of computations carried out for the data listed above.

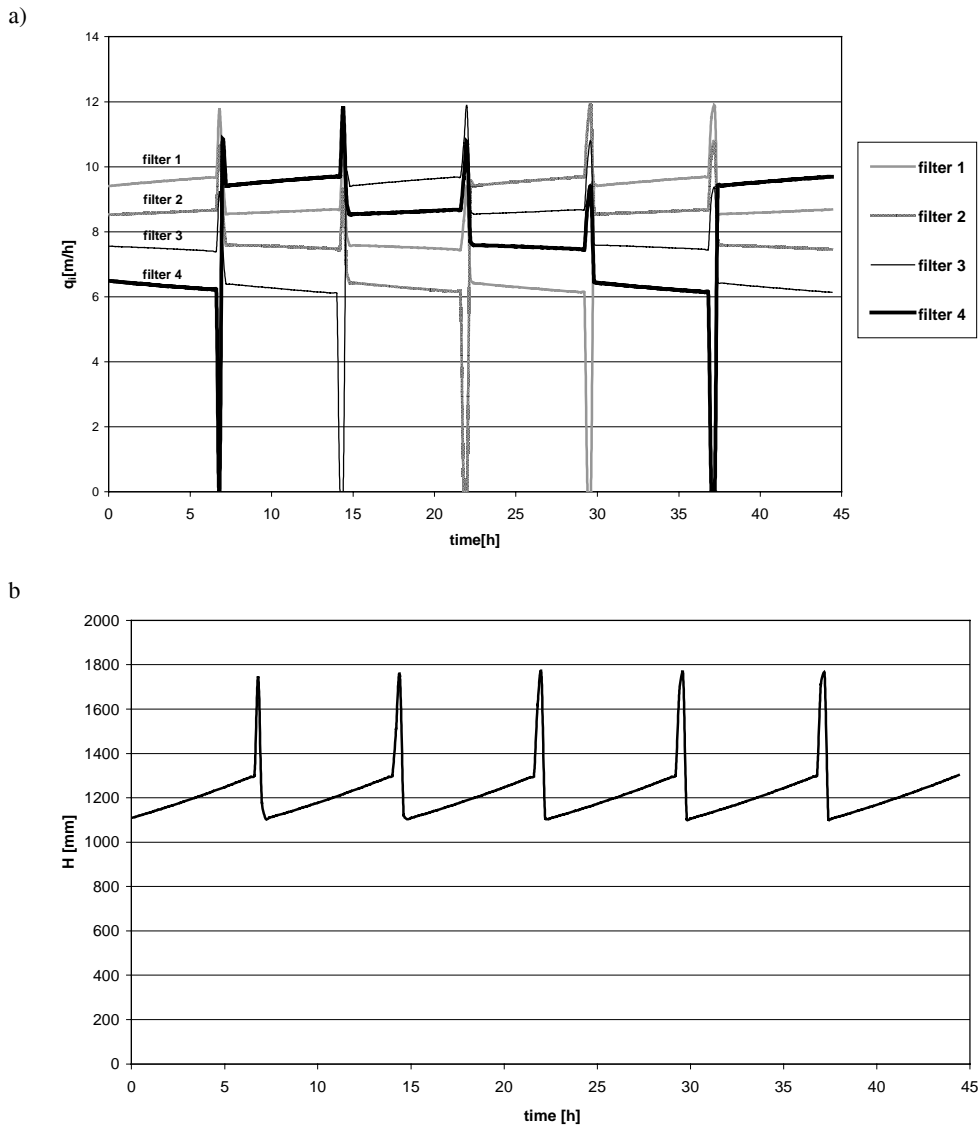


Fig. 1. The examples of flow rates (a) and of the head loss pattern (b) for a four VDR filter plant

DĄBROWSKI [7] using DI BERNARDO's [6], [10] model made the simplification that just after the backwash of the most clogged filter the resistance of grain media of all others working filters was exactly the same as just before this backwash. That is, it was assumed that the clogging that occurred while one filter was taken out for backwashing was negligible. The computer simulation model used here makes no such assumption, and deposition that occurs in the remaining filters during backwashing is fully accounted

for. It was assumed that the time between a filter being taken out for backwashing and brought back into service was 20 minutes. During this time accumulation of water above the filters rises, resulting in increased flow rates through all other filters.

## 5. RESULTS

DĄBROWSKI [7] deduced that flow rates through filters approximately follow the geometric progression defined by the following equation:

$$q_{i+1} = q_1 \left( 1 - \frac{h_0}{H} \right). \quad (6)$$

He proved that the same plant capacity and the same  $q_1/q_{avr}$  can be maintained for different values of total head loss  $H$  by the following procedure:

1. Adjust  $h_0$  to keep the same value of the  $h_0/H$  ratio in equation (6).
2. Calculate  $c_2$  from equation (7) describing head loss  $H - h_0$  through a freshly backwashed filter:

$$H - h_0 = c_1 q_1 + c_2 q_1^n, \quad (7)$$

where  $c_1$  is a constant parameter describing the hydraulic media resistance of a clean filter.

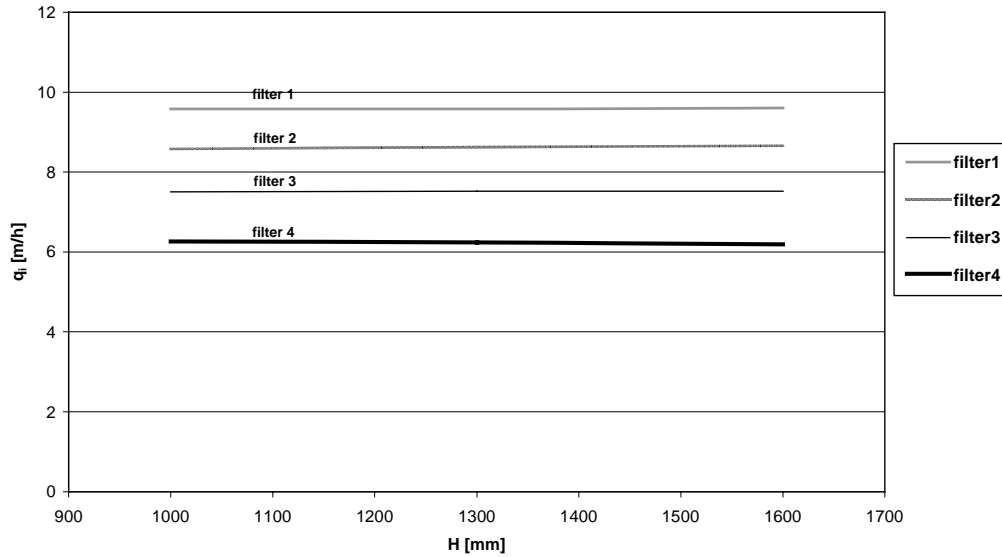


Fig. 2. Constant flow rates  $q_i$  through filters versus  $H$  computed according to the UBE model by MACKIE and ZHAO [13]



The second step of calculations ensures that  $q_1$  is unaffected by any change in  $H$ , and the first that the flow rates  $q_2 \dots q_i \dots q_z$  will also be unaffected. The efficacy of this procedure depends on the accuracy of the approximate equation (6). The validity of this procedure was tested using the computational model applied to four values of the ratio of  $q_1/q_{avr} = 1.10, 1.15, 1.20, 1.25,$  and  $1.30$ . First, new values of  $h_0$  and  $c_2$  were initially predicted according to the procedure described by formulae (6) and (7) and then adjusted by trial-and-error method to ensure the required value of  $H$ . Figure 2 shows the values of  $q_1, q_2, q_3,$  and  $q_4$ , the flow rates through the four filters, for fifty different values of  $H$ , and it can be seen that the values are virtually constant. Thus Dąbrowski's rule is validated. Figure 3 shows the value of  $h_0/H$  for each of the simulations, and these values are again almost constant.

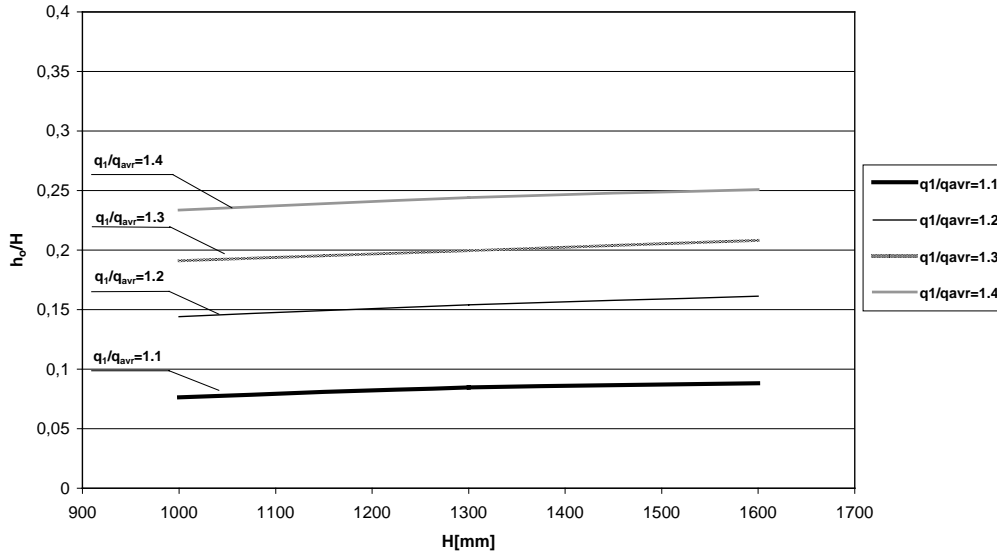


Fig. 3. The values of  $h_0/H$  as a function of  $H$ , satisfying the constraint  $q_1/q_{avr} = \text{constant}$  in figure 2

Equation (7) can be rewritten in the following form

$$H \left( 1 - \frac{h_0}{H} \right) = c_1 q_1 + c_2 q_1^n. \quad (7')$$

Remembering that  $h_0/H$  is constant, this implies that there is a linear relationship between  $H$  and  $c_2$ . Figure 4 shows the relationship between  $H$  and  $c_2$  obtained from the numerical simulations, and the relationship is indeed linear.

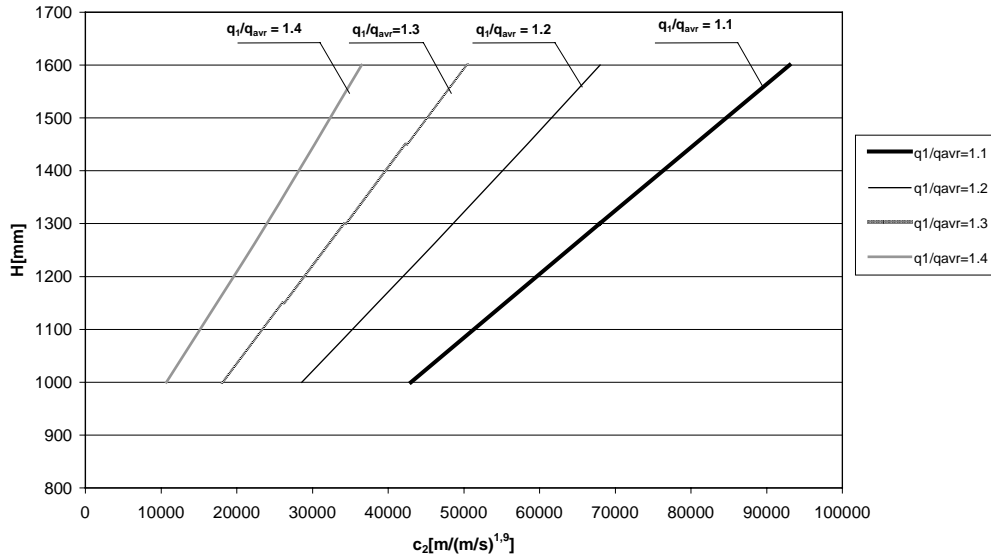


Fig. 4. Linear relationship between  $H$  and  $c_2$  according to the UBE model

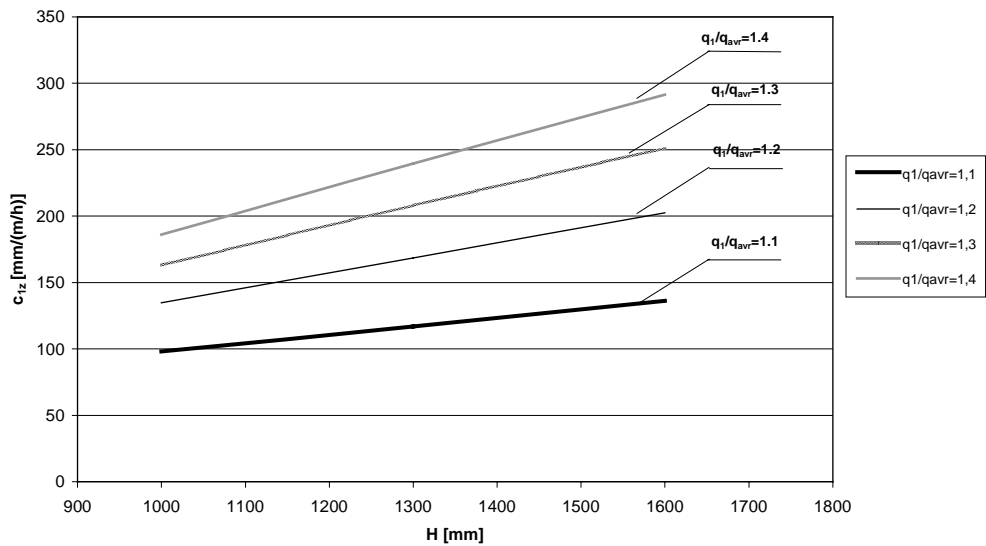


Fig. 5. The resistance of the dirtiest filter media  $c_{1z}$  as a function of  $H$  and  $q_1/q_{avr}$

It seems reasonable to assume that the largest filtrate volume produced from a filter  $z$  is obtained by allowing it to reach as high a hydraulic resistance as possible before a backwash. From DI BERNARDO's model [6], [10] of VDR filters DĄBROWSKI [7] deduced that the highest resistance  $c_{1z}$  of the filter  $z$  just before its backwash (de-

fined by equation (8)) is reached when simultaneously the total head loss  $H$  and the ratio of  $q_1/q_{avr}$  take as large value as possible:

$$H = c_{1z}q_z + c_2q_z^n. \quad (8)$$

Results of numerical tests were shown in figure 5 from which it can be seen that the resistance  $c_{1z}$  of the most clogged filter media before the backwash was higher for any fixed value of  $H$  for higher ratio of  $q_1/q_{avr}$ , and similarly for any fixed  $q_1/q_{avr}$  higher for larger  $H$ . This supports the previous expectations.

VDR and constant rate (CR) operation were compared using the UBE numerical model. Figure 6 shows the ratio of the hydraulic resistance of the most clogged filter media for both modes of operation. VDR mode was simulated for several values of  $q_1/q_{avr}$ . It was assumed that filters were backwashed when the same arbitrarily chosen value of  $H$  was reached. For CR plant operation a head loss created by an open flow rate controller was assumed as equal to 0.2 m. If we assume that higher hydraulic resistance corresponds to higher removal, the VDR operation is more efficient for  $q_1/q_{avr} = 1.2$  or higher which is shown in figure 6. Otherwise it is less efficient than the CR operation.

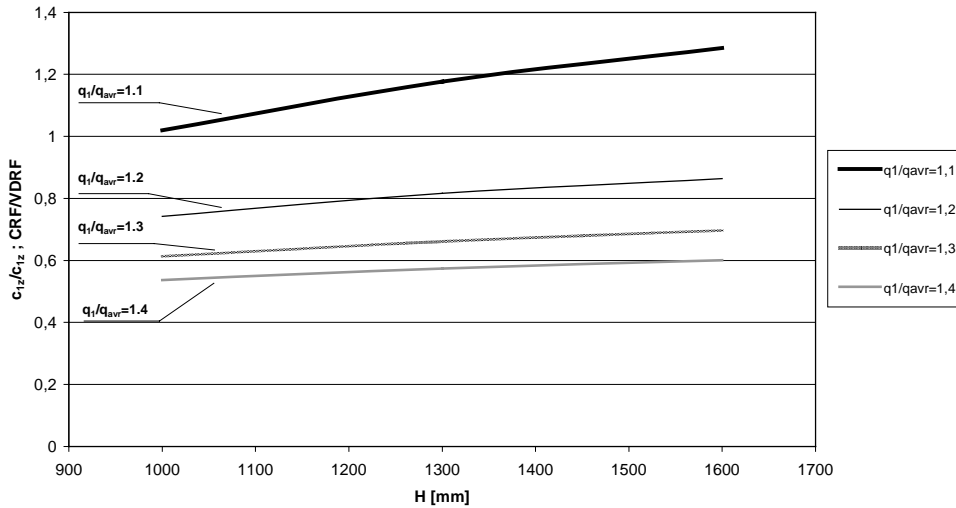


Fig. 6. The ratio of the dirtiest filter media resistance to flow just before a backwash computed once for CRF and once for VDRF control system ( $c_{1z}(\text{CRF})/c_{1z}(\text{VDRF})$ )

## 6. CONCLUSIONS

1. Numerical simulation of VDR filters by a UBE method has given the results confirming earlier predictions by DĄBROWSKI [7] who assumed that the flow through filters forms a geometric progression.

2. The simple rule for adjusting filter operation parameters in response to changes in total head loss  $H$  before a backwash was supported by the numerical simulations.
3. The numerical simulation predicted a linear relationship between  $c_2$  and  $H$  when both were subjected to change in such a way as to keep the same values of  $q_1 \dots q_i \dots q_z$ .
4. The simulation results confirmed the expectations that hydraulic resistance immediately prior to backwash is maximised by allowing  $H$  and  $q_1/q_{avr}$  to be as large as possible.

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#### BADANIA OPTYMALIZACJI FILTRÓW POSPIESZNYCH EKSPLOATOWANYCH ZE SKOKOWO ZMIENNĄ WYDAJNOŚCIĄ

Skonstruowano dynamiczny model umożliwiający szczegółową analizę pracy filtrów pospiesznych eksploatowanych ze skokowo zmienną wydajnością (VDRF). Nowy model uzyskano, łącząc równania hydraulicznego opisu pracy filtrów VDRF z równaniami modelu filtracji wgłębnej, który uwzględnia wpływ przyrostu osadu na straty hydrauliczne i efektywność usuwania zawiesin. Tak opracowany model posłużył do przeprowadzenia szczegółowej analizy pracy filtrów VDRF. Uzyskane wyniki potwierdziły opracowane wcześniej na podstawie uproszczonej analizy modeli statycznych reguły optymalnego doboru parametrów pracy filtrów VDRF. Najważniejsza z tych reguł dotyczy zasady, zgodnie z którą cykl filtracji jest najdłuższy, gdy iloraz maksymalnej i średniej prędkości filtracji oraz straty hydrauliczne w filtrze uzyskują równocześnie wartości maksymalne.