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SYSTEM-STRUCTURAL APPROACH TO HEAT-EXCHANGE PROCESSES IN A SOLAR COLLECTOR

To simulate adequately the process of heat exchange taking place in a solar collector, structural scheme of heat transfer in the collector wall has been developed. Structural model of heat exchange, which is visualization of the interaction and transformation of heat flows both on the boundary and inside a plate, has been built based on the solution of the task of thermal conductivity of the plate with boundary conditions of the third type and due account of internal sources of heat induced by IR irradiation. The model makes it possible to identify thermal-physical parameters.

NOTATION

a – thermal diffusivity, m^2/s;
c – specific heat capacity of the wall, $J/(kg \cdot K)$;
q – heat flux density W/m^2;
R – half of the plate thickness, m;
$T(x, t)$ – temperature field of the plate, K;
x – current coordinate, m;
α – heat transfer coefficient, $W/(m^2 K)$;
θ – air temperature, K;
λ – thermal conductivity of the plate, $W/(m \cdot K)$;
ρ – material density, kg/m^3;
t – time, s.

Nowadays the studies on the usage of ecologically safe and renewable sources of energy are of prime importance. Transformation of sun energy into thermal energy seems to be the most promising trend. Designing and optimization of the solar col-

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lector performance must be based on scientific methods of calculating heat-interchange processes taking place in the apparatus.

We had configured mathematical and physical models of heat exchange in a solar collector before [1]. In order to simulate adequately the process of heat transport in the collector and to identify unknown thermophysical parameters, modern methods of system-structural analysis should be employed. Structural models and methods of heat transport analysis are the basis for designing information-measuring systems with elements of various physical origin. These models are the basis for constructing the schemes of identification of a substance thermophysical properties as well as the solution of the tasks enabling optimization of monitoring such processes. The system-structural approach allows us to shorten to a minimum the computer time of the problem solution by developing the model of thermal conductivity and its analysis, the solution of the task of optimization and monitoring of heat transfer which are complicated because of the internal nonstationary sources of heat of various physical origin. The models and methods developed can be applied to calculate and design heat exchangers. The purpose of the present paper is to construct the scheme of heat interchange at the collector wall.

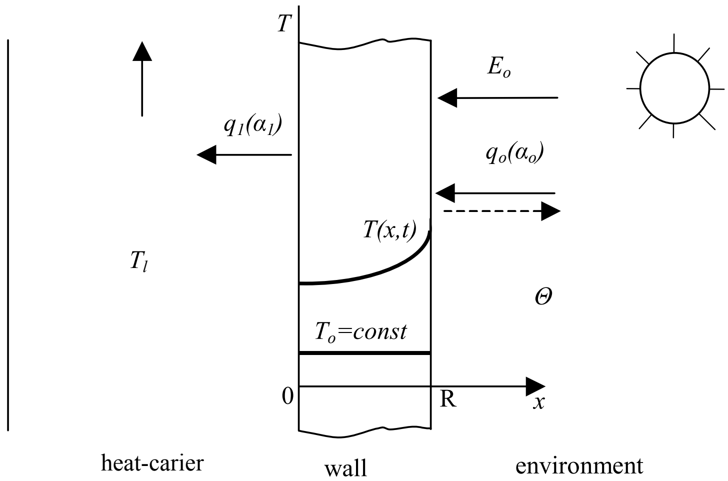

Fig. 1. The scheme of heat interchange in the collector wall

First, we deal with the case where heat carrier consumption does not change. The main problem at this stage is the solution of the task related to warming up of the collector wall. Let us represent a physical model of the process assuming a number of limitations. The collector wall is very thin. Hence, it can be regarded as being a finite plate with the thickness $2R$ and an initial temperature T_0 = const. The temperature Θ of the environment and the temperature T_l of the heat carrier from inside of the wall are constant. The solar energy flux reaches the collector panel. Density of the radiant energy E_0 is constant (figure 1). One can think that these conditions can be obeyed when the time intervals Δt are short. The time to be calculated was divided into still shorter intervals. The final conditions of the previous time interval could be regarded as an initial condition of the interval Δt. Similarly, the changes of the process parameters (such as flow density of solar energy, heat transport coefficients for liquid and gaseous phases and so on) with time can be approximated as segment stagnant functions that can be regarded as being constant for the interval involved, but they change after transition to the following time interval.

We are of the opinion that adsorption of radiant energy by the wall follows the Buger law [2]. According to this law, internal sources of heat appear in the wall, and their intensity decreases in the directions the irradiation is spreading

$$q(x) = \mu(1-\omega)E_0 \exp(-\mu(R-x)),$$

where ω is the index of refraction, μ stands for an attenuation factor, and x is the wall thickness coordinate. A convective heat exchange between bounding surfaces of the wall, environment and the liquid in the collector takes place. It should be found out how the temperature is distributed along the wall thickness and how it changes with time.

Mathematical formulation of the problem is as follows:

$$\frac{\partial T(x,t)}{\partial t} = a\frac{\partial^2 T(x,t)}{\partial x^2} + \frac{\mu}{c\rho}(1-\omega)\cdot E_0 \exp(-\mu(R-x)), \quad t>0,\ 0<x<R, \tag{1}$$

$$T(x,0) = T_0, \tag{2}$$

$$\lambda\frac{\partial T(R,t)}{\partial x} = \alpha_0[\Theta - T(R,t)] + E_0, \tag{3}$$

$$\lambda\frac{\partial T(0,t)}{\partial x} = \alpha_1[T(0,t) - T_l]. \tag{4}$$

We should find $T(x, t)$ at $t>0$, $0 \le x \le R$. Use has been made of an operational method to solve the task of thermal conductivity [3]. We obtained the solution of the task by applying the Laplace transform [1]. Let $L[T(x, t)] = T(x, s)$.

The solution of the problem of thermal conductivity of solar collector plate by means of the above method has an independent value. It allows us to carry out without mistake a thermophysical experiment and to optimize heating of the plate by the monitoring of incoming parameters. Being modern and powerful tool of investigation, the system-structural approach enables all these tasks to be efficiently treated. Let us develop a structural scheme of the task solution (1–4) according to the system-structural analysis method. Let the solution of a differential equation of thermal conductivity in the space of the Laplace representation be treated as the system represented by a corresponding structural scheme. The elements of the latter are mathematical statements (transfer functions) that lay down the rules for transformations of heat effects on the object into the reaction initiated by them. Operation functions describing the segments of the scheme are not complex in the form. Its incomings and outcomings have physical interpretation. The scheme constructed in such a way will be visualization of the interaction and transformation of heat flow both on the boundary and inside the plate. In fact, it is a mathematical model of the process. For convenience, let us represent the temperatures $T(x, t)$ and T_l in the form of sums: $T(x,t) = T_0 + \Delta T(x,t)$; $\Theta = T_0 + \Delta\Theta$; $T_l = T_0 + \Delta T_l$.

Then the solution $\Delta\overline{T}(x,s)$ of the differential equation of thermal conductivity in the Laplace transform space can be expressed by $\Delta\overline{T}(0,s)$ and $\Delta\overline{T}(R,s)$:

$$\Delta\overline{T}(x,s) = \Delta\overline{T}_l ch\sqrt{s/a}x + sh\sqrt{\frac{s}{a}}x \cdot \left\{\frac{\Delta\overline{T}(R,s) - \Delta\overline{T}(0,s)ch\sqrt{s/a}R}{sh\sqrt{s/a}R}\right.$$

$$+\frac{\frac{\mu(1-\omega)}{c\gamma a}\frac{E_0}{s}\exp(-\mu R)}{sh\sqrt{s/a}R}\left\{\frac{1}{\mu^2 - s/a}\exp(\mu R) + \frac{1}{2\sqrt{s/a}(\sqrt{s/a}-\mu)}\exp(\sqrt{s/a}R)\right.$$

$$\left.+\frac{1}{2\sqrt{s/a}(\sqrt{s/a}+\mu)}\exp(-\sqrt{s/a}R)\right\} + \left.\frac{1}{2\sqrt{s/a}(\sqrt{s/a}+\mu)}\exp(-\sqrt{s/a}x)\right\}$$

$$-\frac{\frac{\mu(1-\omega)}{c\gamma a}\frac{E_0}{s}\exp(-\mu R)}{sh\sqrt{s/a}R}\left\{\frac{1}{\mu^2 - s/a}\exp(\mu x) + \frac{1}{2\sqrt{s/a}(\sqrt{s/a}-\mu)}\exp(\sqrt{s/a}x)\right.$$

$$\left.+\frac{1}{2\sqrt{s/a}(\sqrt{s/a}+\mu)}\exp(-\sqrt{s/a}x)\right\}, \tag{5}$$

where:

$$D = \frac{\mu}{c\rho a}(1-\omega)E_0\exp(-\mu\cdot R),$$

$$H_1 = \frac{\alpha_0}{\lambda},$$

$$H_2 = \frac{\alpha_1}{\lambda},$$

$\Delta\overline{T}(0,s)$ – the L-representation of an increment in the temperature of the plate at $x = 0$, which can be determined from the formula:

$$\Delta\overline{T}(0,s) = \frac{1}{\alpha_1 + \lambda\sqrt{s/a}\cdot cht\sqrt{s/a}R}$$
$$\cdot\left(\Delta T_l \frac{1}{s}\alpha_1 + \frac{\lambda\sqrt{s/a}}{sh\sqrt{s/a}R}\left(\Delta\overline{T}(R,s) + \frac{\mu(1-\omega)}{c\gamma a}\cdot\frac{E_0}{s}\exp(-\mu\cdot R)\cdot\left(\frac{\exp(\mu R)}{\mu^2 - s/a}\right.\right.\right.$$
$$\left.\left.\left. + \frac{1}{2\sqrt{s/a}(\sqrt{s/a}-\mu)}\exp(\sqrt{s/a}R) + \frac{1}{2\sqrt{s/a}(\sqrt{s/a}+\mu)}\exp(-\sqrt{s/a}R)\right)\right)\right). \quad (6)$$

$\Delta\overline{T}(R,s)$ is the L-representation of an increment in the temperature of the plate at $x = R$ which can be expressed by $\Delta\overline{T}(0,s)$. By making use of the formal rules of structural scheme transformations, we can represent $\Delta\overline{T}(R,s)$ by the scheme of temperature on the boundary $x = R$. Figure 2 shows the structural scheme to include incoming and outcoming values of physical origin, which means that they can be measured. In a given case, the incoming values are as follows:

$\alpha_o \frac{\Delta\Theta}{s}$ – the segment representing an increment in the component of convection heat flow absorbed by the plate surface of solar collector due to the change in the environment temperature by $\Delta\Theta$;

$\frac{E_0}{s}$ – the segment representing the component of solar radiation flow absorbed by the plate surface;

$\frac{\Delta T_l}{s}$ – an increment in the segment representing convective heat flow from an internal surface of the plate of SC to a heat carrier when the temperature of the latter changes by ΔT_l.

The structural scheme (figure 2) presents intermediate coordinates $\Delta\overline{T}(R,s)$, $\Delta\overline{T}(0,s)$, $\Delta\overline{q}(0,s)$. One can take these into consideration either while optimizing the process of heat interchange in solar collector or while constructing the schemes of thermophysical parameter identification. Thus, the value of the coefficients of heat transfer from the internal side of the wall to the heat carrier can be found by determining the temperatures of the surface $\Delta T(0,t)$, of the environment $\Delta\Theta$ and heat carrier ΔT_l as well as solar energy flow density E_0.

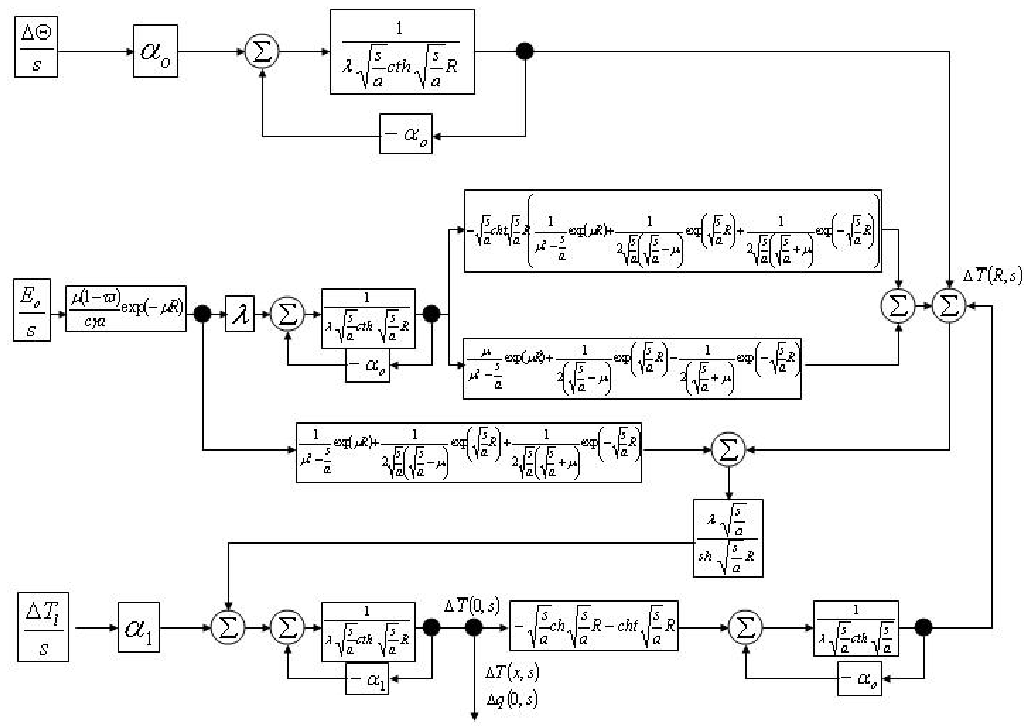

Fig. 2. Structural scheme of the solution of the equation for the plate thermal conduction

The experiment carried out properly allows us to construct quadratic functional being the difference between the temperatures measured and calculated based on the mathematical model temperature of the surface of the plate. The value of the coefficient α_1 can be determined in terms of minimum of medium-quadratic functional.

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PROCESY WYMIANY CIEPŁA W KOLEKTORZE SŁONECZNYM

Aby symulować proces wymiany ciepła w kolektorze słonecznym, opracowano strukturalny schemat przenoszenia ciepła. Strukturalny model wymiany ciepła, będący wizualizacją wzajemnych oddziaływań strumieni ciepła zarówno na granicy, jak i wewnątrz płyty kolektora, został skonstruowany na podstawie parametrów przewodności cieplnej płyty z uwzględnieniem wewnętrznego źródła ciepła indukowanego promieniowaniem podczerwonym. Model umożliwia identyfikację parametrów termiczno-fizycznych.