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PHYSICAL AND MATHEMATICAL MODEL OF GRAVITATION-PUMP RESERVOIR IN SEWAGE SYSTEM

Effective methods of rainwater and combined sewage drainage from urbanized areas to storage reservoirs are presented. Theoretical fundamentals of wastewater accumulation based on authors' designs of storage reservoirs with the gravitation-pump arrangement of accumulation chambers are given. A physical and mathematical model that enables a description of hydraulic processes taking place in reservoirs of this type was constructed.

DESIGNATIONS

c – rain frequency, once per *c* year(s);
Fz – surface of drainage area, ha;
F_{KAG} – horizontal surface of the gravitational accumulation chamber, m^2;
F_{KP} – horizontal surface of through-flow chamber, m^2;
fo – cross-sectional area of the outflow channel opening, m^2;
g – acceleration due to gravity, m/s^2;
H – mean annual precipitation, mm;
h – height of wastewater that fills the through-flow chamber *KP* of the storage reservoir from the level of outflow pipe axis as the reference, m;
h_{aw} – elevation of emergency overfall edge in the through-flow chamber *KP* of the storage reservoir from the level of outflow pipe axis as the reference, m;
Hp – height of wastewater that fills the gravitational accumulation chamber *KAG* of the storage reservoir from the level of outflow pipe axis as the reference, m;
Hpw – mean height of wastewater that fills the accumulation chamber *KAW* from the level of the chamber bottom as the reference, m;
hr – mean elevation of the bottom of the gravitational accumulation chamber from to the level of outflow pipe axis as the reference, m;

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QB – momentary wastewater flow intensity for $Td = Tp$ within the time range of $t \in \langle 0, Tp)$, m^3/s;
QB' – momentary wastewater flow intensity for $Td > Tp$ within the time range of $t \in \langle 0, Tp)$, m^3/s;
QB'' – momentary wastewater flow intensity for $Td < Tp$ within the time range of $t \in \langle 0, Td)$, m^3/s;
QC' – momentary wastewater flow intensity for $Td > Tp$ within the time range of $t \in \langle 0, Td)$, m^3/s;
QC'' – momentary wastewater flow intensity for $Td < Tp$ within the time range of $t \in \langle 0, Td)$, m^3/s;
QD – momentary wastewater flow intensity for $Td = Tp$ within the time range of $t \in \langle Tp, Tp+Td)$, m^3/s;
QD' – momentary wastewater flow intensity for $Td > Tp$ within the time range of $t \in \langle Tp, Tp+Td)$, m^3/s;
QD'' – momentary wastewater flow intensity for $Td < Tp$ within the time range of $t \in \langle Tp, Tp+Td)$, m^3/s;
Td – rain duration time, min;
tk – the time of area rainfall flow concentration, min;
Tp – the time of rainfall water drainage; the time necessary for sewage network designing based on the method of boundary intensities, min;
tp – the time of wastewater flow through the sewage system from the farthest point of the drainage area to the computational cross-section of the channel, min;
tr – the time of wastewater retention in the channel, min;
μ – flow rate coefficient at reservoir outflow opening;
μ_1 – flow rate coefficient at non-submerged inter-chamber overfall;
μ_2 – flow rate coefficient at non-submerged inter-chamber overfall for a non-submerged layer of overflowing wastewater;
μ_3 – flow rate coefficient at submerged inter-chamber overfall for a submerged layer of overflowing wastewater.

1. INTRODUCTION

In modern rainwater and combined drainage and utilization systems, the effective methods of regulation of flow intensities, both in the stage of their flow through sewage system and before entering wastewater treatment plants, should be applied. A hydraulic overload has an unfavourable effect on sewage systems as it causes a frequent occurrence of pressurized flows, overloading of sewage facilities and units with possible wastewater penetration into the ground. The supply of precipitation wastewaters to wastewater treatment plant(s) requires the use of storm overflows that discharge some wastewater beyond the sewage system to water-receiving body, which brings about the unfavourable consequences in terms of the purity of surface waters.

That problem may be solved if use is made of single and multiple-chamber storage reservoirs. In those reservoirs, wastewater is accumulated due to operation of hydraulic, gravitational [1], gravitation-vacuum [2] or gravitation-pump [3] units in accumulation chambers. The above mentioned facilities are characterized by a specific range of economically-effective applications of sewage-system as well as by specific local hydraulic conditions of sewage systems. In gravitational reservoirs, considerable hydraulic heads (differences of levels) and large surface areas for their installing are indispensable. In alternative gravitation-vacuum reservoirs, these conditions can partially be obeyed but, on the other hand, such reservoirs require expensive gas-tight walls and roofs. Taking account of the above, we propose new designs of reservoirs that are based on gravitation-pump arrangements of accumulation chambers. They permit highly effective wastewater storage in open chambers with theoretically unlimited levels of wastewater being stored.

The research undertaken in order to create new designs of storage reservoirs allowed us to develop scientific foundations for eighteen new gravitation-pump reservoirs of different hydraulic characteristics, which are useful in specific local and investment-related conditions. Because of the pump chamber localization, three basic types of designs are identified:

- gravitation-pump reservoirs equipped with an upper accumulation chamber located above the gravitational accumulation chamber,
- gravitation-pump reservoirs equipped with a lower pump accumulation chamber below the gravitational accumulation chamber,
- hybrid reservoirs with pump chambers located below and above the gravitational accumulation chamber.

2. HYDRAULIC MODEL OF *GPW* TYPE RESERVOIR

A basic design of gravitation-pump reservoir with upper accumulation chamber is the reservoir of the *GPW* type. The idea of its hydraulic system consists in adapting the known *Contract*-type two-chamber reservoir by equipping it with a chamber with pump and a filling and emptying system. Figure 1 presents the hydraulic system and the directions of wastewater flow between the chambers during wastewater accumulation.

Due to an increasing rate of wastewater afflux to the reservoir, the through-flow chamber is filled, and then, on exceeding the level of interchamber overfall edge, the wastewater excess is carried away to the gravitational-accumulation chamber. If the wastewater inflow continues and a hydraulic capacity of the outflow channel from the reservoir is exceeded, the storage space of the gravitational-accumulation chamber becomes filled to its capacity. At that moment the pump-conveyance system is switched on and the upper accumulation chamber is being filled. Theoretically, this process lasts until the latter chamber is filled to its capacity, unless the wastewater inflow to the reservoir is reduced below the level established for a decreased wastewater outflow from the reservoir.

The analysis of the operation of the reservoir of this type permitted identification of two ways of its emptying, which differ in the sequence of wastewater outflow from particular reservoir chambers. The sequence of such an emptying determines the hydraulic processes in the reservoir and has a decisive effect on the function representing the intensity of wastewater outflow from the reservoir. Thus, the selection of the way of the reservoir emptying version should be preceded by the analysis of the storage reservoir effect on other elements of the sewage system that are located below this reservoir.

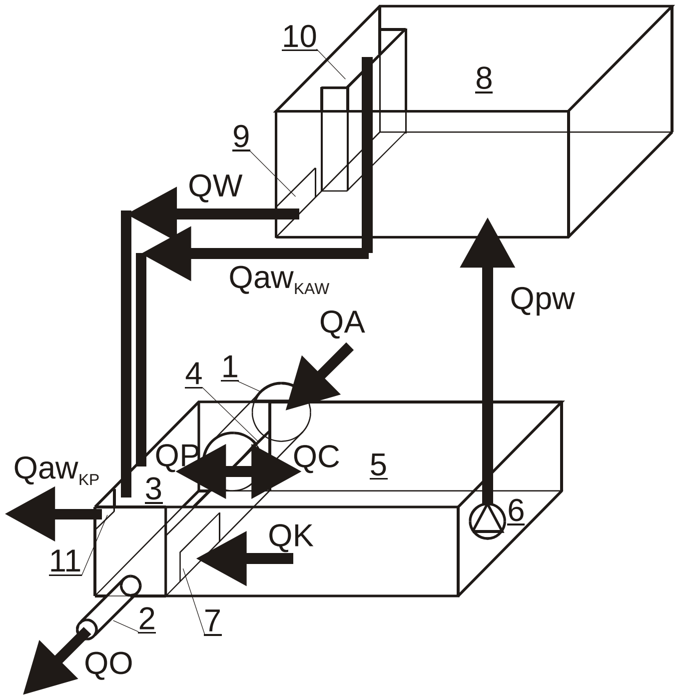

Fig. 1. Hydraulic system of gravitation-pump reservoir of the *GPW* type and the directions of wastewater flow between chambers: *QA* – wastewater inflow into reservoir, *QO* – wastewater outflow from reservoir, *QC* – wastewater flow from the through-flow chamber, interchamber overfall to the accumulation chamber, *QP* – wastewater flow from the accumulation chamber, interchamber overfall to the through-flow chamber, *QK* – wastewater flow through the hole of the non-return flap valve, *Qpw* – wastewater flow forced by pump-system operation, *QW* – wastewater outflow from the bottom sluice in the accumulation chamber to the through-flow chamber, Qaw_{KP} – wastewater outflow from the emergency overfall of the gravitational section of the reservoir, Qaw_{KAW} – wastewater outflow from the emergency overfall of the accumulation chamber, 1 – inflow channel, 2 – outflow channel, 3 –through-flow chamber, 4 – interchamber overfall, 5 – gravitational accumulation chamber, 6 – pumping unit, 7 – non-return flap valve, 8 – upper accumulation chamber with pump, 10 – emergency overfall of accumulation chamber, 11 – emergency overfall of gravitational chambers

Within the framework of a theoretical description of the method of the *GPW* reservoir operation in the sewage network system, twenty eight specific phases, constituting a closed cycle of its operation, were identified. For each of those phases hydraulic boundary conditions were formulated. They refer to the flows and the characteristic wastewater levels in the reservoir.

3. HYDROGRAMS OF RAIN- AND WASTEWATER INFLOW TO RESERVOIR

In order to develop a mathematical model of storage-reservoir operation in sewage network system function representing the wastewater inflow to the reservoir should be derived. There are known methods [4] of describing each hydrogram of wastewater flow in sewage network system by means of the sequence of linear functions that approximate each arbitrarily assumed function representing the variability of wastewater flow. From relationship (1) expressed by the equation of straight line passing through two points, it is possible to determine the intensity of wastewater flow at each time t based either on the known intervals of variability of the wastewater flow or on the angle of inclination of the straight lines representing individual variability intervals of that function.

$$QA(t) = (QA_{i+1} - QA_i)(t - t_i)(t_{i+1} - t_i)^{-1} + QA_i. \tag{1}$$

Because of a common application of the method of limiting intensities in the sewage network design, the rain times Td and the curves of the variability of the intensities of wastewater flow through sewage network system, formulated for those times, in newly developed mathematical model were referred to the computational hydrogram, which is compatible with that method and related to the time Tp of wastewater flow described by relationship (2). The time Tp is a sunm of the time necessary for the concentration of the surface-area drainage and the time of wastewater retention in the channel:

$$Tp = tp + tk + tr. \tag{2}$$

Figure 2 presents characteristic hydrograms of wastewater flow through the sewage network system, depending on the length of the rain duration Td.

Depending on the relationships between the times Td and Tp, mathematical relationships were derived, which permit determination of the intensity of wastewater inflow to reservoir at each time t in different intervals of variability of the wastewater flow functions.

1. Rain time equal to inflow time, $Td = Tp$ (figure 2a)

- for the increasing function:

$$QB = QA \cdot Tp^{-1} \cdot t, \tag{3}$$

- for the decreasing function:

$$QD = QA \cdot (1 + Td \cdot Tp^{-1}) - QA \cdot Tp^{-1} \cdot t = 2 \cdot QA - QA \cdot Tp^{-1} \cdot t. \tag{4}$$

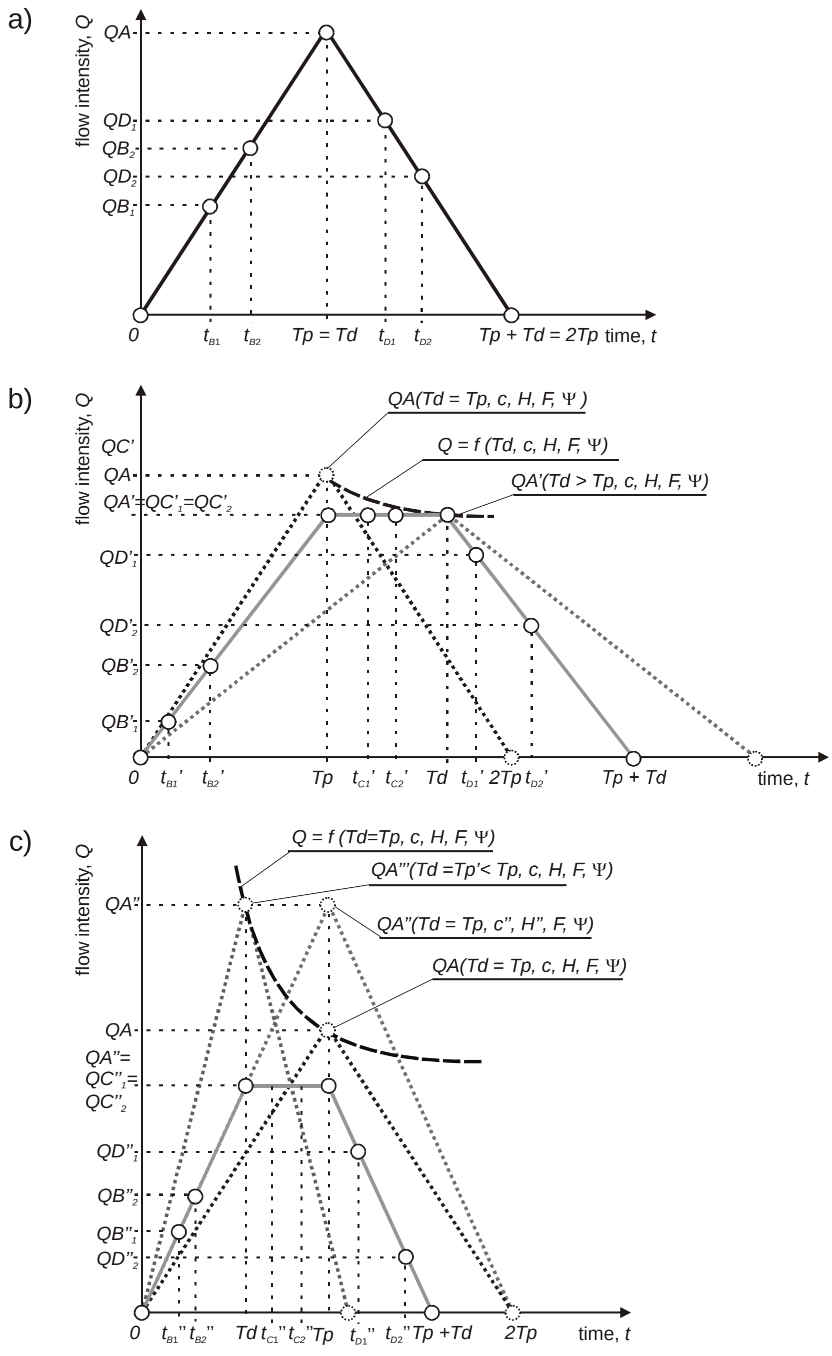

Fig. 2. Computational hydrograms of wastewater flow through sewage network system at different rain times: a) $Td = Tp$, b) $Td > Tp$, c) $Td < Tp$

2. Rain time longer than the inflow time, $Td > Tp$ (figure 2b)
- for the increasing function:

$$QB' = QA' \cdot Tp^{-1} \cdot t\,, \tag{5}$$

- for the constant function:

$$QC' = QA', \tag{6}$$

- for the decreasing function:

$$QD' = QA' \cdot (1 + Td \cdot Tp^{-1}) - QA' \cdot Tp^{-1} \cdot t.. \tag{7}$$

3. Rain time shorter than the inflow time, $Td < Tp$ (figure 2c)
- for the increasing function:

$$QB'' = QA'' \cdot Tp^{-1} \cdot t\,, \tag{8}$$

- for the constant function:

$$QC'' = QA'' \cdot Td \cdot Tp^{-1}, \tag{9}$$

- for the decreasing function:

$$QD'' = QA \cdot (1 + Td \cdot Tp^{-1}) - QA \cdot Tp^{-1} \cdot t, , \tag{10}$$

where:

$$QA = F \cdot \Psi \cdot q_{dm} = 6.631 \cdot H^{2/3} \cdot c^{1/3} \cdot Tp^{-2/3} \cdot F \cdot \Psi, \tag{11}$$

$$QA' = F \cdot \Psi \cdot q_{dm}(Td) = 6.631 \cdot H^{2/3} \cdot c^{1/3} \cdot Td^{-2/3} \cdot F \cdot \Psi, \tag{12}$$

$$QA'' = F \cdot \Psi \cdot q_{dm}(Tp) = 6.631 \cdot H^{2/3} \cdot c^{1/3} \cdot Tp^{-2/3} \cdot F \cdot \Psi. \tag{13}$$

4. MATHEMATICAL MODEL OF RESERVOIR OPERATION

The developed hydraulic model of the reservoir operation is the basis for formulating relationships determining the balance of wastewaters in individual chambers of the reservoir in all phases of its operation while accounting for variable boundary conditions. The model of wastewater balance was developed using the mass conservation law illustrated by figure 3 for gravitation-pump reservoir of the *GPW* type.

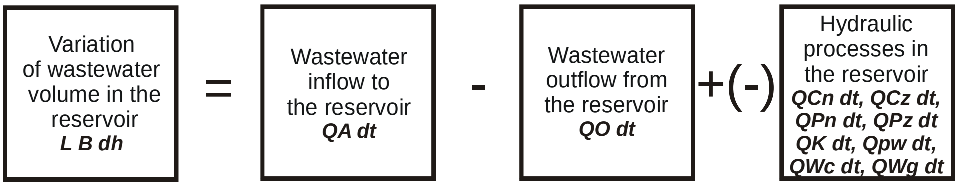

Fig. 3. Diagram of wastewater balance elements in gravitation-pump reservoir with an upper accumulation chamber

Based on the equations of wastewater balance and the curves representing the wastewater flow through sewage network system, a detailed mathematical model of the reservoir has been developed in the form of differential equations and the systems of equations describing wastewater filling increments in individual chambers of the reservoir in a dynamical system. The mathematical model takes into account the wastewater flow between the chambers, and in particular the hydraulic conditions of the interchamber overfall operation, the character of wastewater outflow from a bottom sluice of the upper accumulation chamber, as well as the operation of the pumping system that allows the upper accumulation chamber to be filled. A part of the mathematical model describing two selected phases of storage reservoir operation in the sewage network system formulated for arbitrary *Td* time of rain, is presented.

1. Filling phase of the through-flow chamber of storage reservoir (figure 4)

Hydraulic boundary conditions in the filling range: $0 < h \le h_{aw}$, $Hp = hr$, $Hpw = 0$

- under inflow conditions: $QB_1 < QB_2$, $QB_1' < QB_2'$, $QB_1'' < QB_2''$

$$\frac{dh}{dt} = QA \cdot F_{KP}^{-1} \cdot Tp^{-1} \cdot t - 1.41 \cdot g^{1/2} \cdot \mu \cdot fo \cdot F_{KP}^{-1} \cdot h^{1/2}, \tag{14}$$

- under inflow conditions: $QC_1 = QC_2$, $QC_1' = QC_2'$

$$\frac{dh}{dt} = QA \cdot F_{KP}^{-1} - 1.41 \cdot g^{1/2} \cdot \mu \cdot fo \cdot F_{KP}^{-1} \cdot h^{1/3}, \tag{15}$$

- under inflow conditions: $QC_1'' = QC_2''$

$$\frac{dh}{dt} = QA \cdot Td \cdot Tp^{-1} \cdot F_{KP}^{-1} - 1.41 \cdot g^{1/2} \cdot \mu \cdot fo \cdot F_{KP}^{-1} \cdot h^{1/2}, \tag{16}$$

- under inflow conditions: $QD_1 > QD_2$, $QD_1' > QD_2'$, $QD_1'' > QD_2''$

$$\frac{dh}{dt} = QA \cdot F_{KP}^{-1} \cdot (1 + Td \cdot Tp^{-1}) - QA \cdot F_{KP}^{-1} \cdot Tp^{-1} \cdot t - 1.41 \cdot g^{1/2} \cdot \mu \cdot fo \cdot F_{KP}^{-1} \cdot h^{1/2}. \tag{17}$$

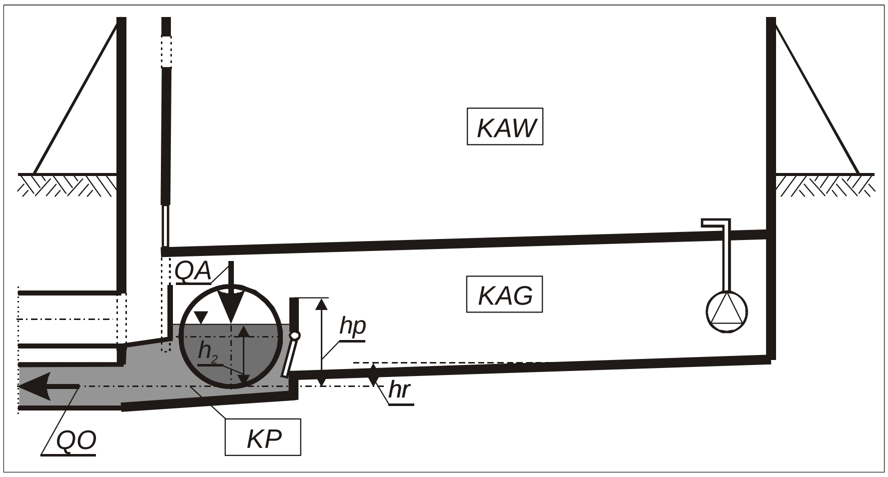

Fig. 4. Hydraulic conditions in the filling phase of the through-flow chamber in the reservoir

2. Filling phase of the gravitational accumulation chamber through interchamber overfall with hydraulic function of immersed overfall (figure 5)

Hydraulic boundary conditions in the filling range: hp < h ≤ h_{aw}, h > Hp, hp < Hp < ho

$$\begin{cases} \dfrac{dh}{dt} = \begin{cases} \text{for inflow conditions: } QB_1 < QB_2, QB_1' < QB_2', QB_2'' < QB_2'' \\ QA \cdot F_{KP}^{-1} \cdot Tp^{-1} \cdot t - 1.41 \cdot g^{1/2} \cdot \mu \cdot fo \cdot F_{KP}^{-1} \cdot h^{1/2} - 0.943 \cdot g^{1/2} \cdot b \cdot \mu_2 \\ F_{KP}^{-1} \cdot (h - Hp)^{3/2} - 1.41 \cdot g^{1/2} \cdot b \cdot \mu_3 \cdot F_{KP}^{-1} \cdot (Hp - hp) \cdot (h - Hp)^{1/2}, \\ \text{for inflow conditions: } QC_1 = QC_2, QC_1' = QC_2' \\ QA \cdot F_{KP}^{-1} - 1.41 \cdot g^{1/2} \cdot \mu \cdot fo \cdot F_{KP}^{-1} \cdot h^{1/2} \\ - 0.943 \cdot g^{1/2} \cdot b \cdot \mu_2 \cdot F_{KP}^{-1} \cdot (h - Hp)^{3/2} \\ - 1.41 \cdot g^{1/2} \cdot b \cdot \mu_3 \cdot F_{KP}^{-1} \cdot (Hp - hp) \cdot (h - Hp)^{1/2}, \\ \text{for inflow conditions: } QC_1'' = QC_2'' \\ QA \cdot Td \cdot Tp^{-1} \cdot F_{KP}^{-1} - 1.41 \cdot g^{1/2} \cdot \mu \cdot fo \cdot F_{KP}^{-1} \cdot h^{1/2} - 0.943 \cdot g^{1/2} \cdot b \cdot \mu_2 \\ F_{KP}^{-1} \cdot (h - Hp)^{3/2} - 1.41 \cdot g^{1/2} \cdot b \cdot \mu_3 \cdot F_{KP}^{-1} \cdot (Hp - hp) \cdot (h - Hp)^{1/2}, \\ \text{for inflow conditions: } QD_1 < QD_2, QD_1' < QD_2', QD_1'' < QD_2'' \\ QA \cdot F_{KP}^{-1} \cdot (1 + Td \cdot Tp^{-1}) - QA \cdot F_{KP}^{-1} \cdot Tp^{-1} \cdot t - 1.41 \cdot g^{1/2} \cdot \mu \\ fo \cdot F_{KP}^{-1} \cdot h^{1/2} - 0.943 \cdot g^{1/2} \cdot b \cdot \mu_2 \cdot F_{KP}^{-1} \cdot (h - Hp)^{3/2} \\ - 1.41 \cdot g^{1/2} \cdot b \cdot \mu_3 \cdot F_{KP}^{-1} \cdot (Hp - hp) \cdot (h - Hp)^{1/2}, \end{cases} \\ \dfrac{dHp}{dt} = 0.943 \cdot g^{1/2} \cdot b \cdot \mu_2 \cdot F_{KAG}^{-1} \cdot (h - Hp)^{3/2} + 1.41 \cdot g^{1/2} \cdot b \cdot \mu_3 \cdot F_{KAG}^{-1} \cdot (Hp - hp) \cdot (h - Hp)^{1/2}. \end{cases} \quad (18)$$

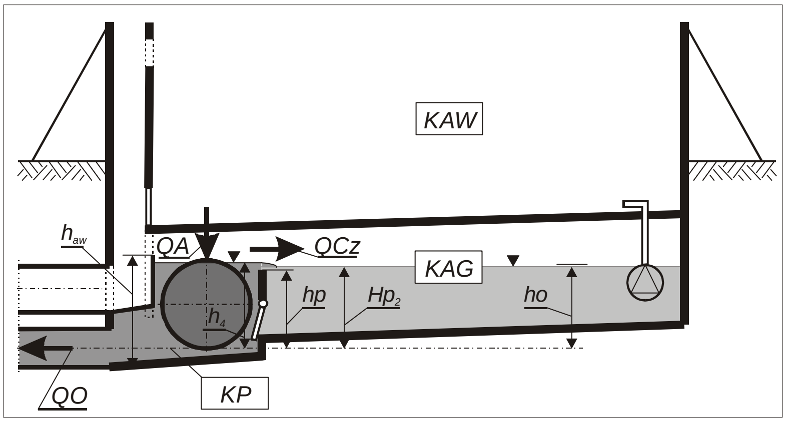

Fig. 5. Hydraulic conditions in the filling phase of the gravitational accumulation chamber through the interchamber overfall with hydraulic function as immersed overfall

5. SUMMARY

Cost of building sewage systems, including storage reservoirs, is a complex function, affected by many different and interdependent factors. Economically satisfactory results are achieved, among others, due to a rational application of a suitable design of storage reservoirs, its role in the sewage system and the wastewater flow reduction. In order to optimize the sewage system, it is important to develop new concepts of reservoirs that broaden the economically viable wastewater storage in reservoirs.

The designs of new storage reservoirs of the *GPW* type that operate in gravitation--pump systems, whose general theoretical foundations are presented in this paper, enable an effective control of wastewater flow due to their multichamber structure. The utilization of chambers with pumps unrestrained shaping of their hydraulic systems, depending on local investment conditions.

On the basis of the detailed model of the storage reservoir of the *GPW* type, new numerical *WEGA* software programs were developed for the simulation of operation of reservoirs of that type in the course of rain- and wastewater inflow. The programs enable us to study the variations in filling dynamics in all chambers of the reservoir at the same time, to study the intensity of wastewater outflow and to define essential project parameters of reservoirs.

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MODEL FIZYCZNY I MATEMATYCZNY DZIAŁANIA ZBIORNIKA GRAWIATCYJNO-POMPOWEGO W KANALIZACJI

Zbiorniki retencyjne są elementami nowoczesnych lub modernizowanych systemów kanalizacyjnych i służą do regulacji ilości oraz jakości ścieków zarówno na etapie ich transportu sieciami kanalizacyjnymi, jak i przed oczyszczalniami. Przedstawiono wyniki badań teoretycznych nad autorskimi projektami zbiorników retencyjnych funkcjonujących w grawitacyjno-pompowych układach komór akumulacyjnych. Pokazano sposób budowania modelu hydraulicznego zbiorników typu *GPW* oraz modelu matematycznego stanowiącego podstawę do opracowania programów symulacji numerycznej działania tego typu zbiorników w sieci kanalizacyjnej.